IberoAmerican 2018

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–	Day 1
1	For each integer $n \geq 2$, find all integer solutions of the following system of equations: $$x_1 = (x_2 + x_3 + x_4 + ... + x_n)^{2018}$$ $$x_2 = (x_1 + x_3 + x_4 + ... + x_n)^{2018}$$ $$\vdots$$ $$x_n = (x_1 + x_2 + x_3 + ... + x_{n-1})^{2018}$$
2	Let ABC be a triangle such that $\angle BAC = 90^\circ$ and $AB = AC$. Let M be the midpoint of BC. A point $D \neq A$ is chosen on the semicircle with diameter BC that contains A. The circumcircle of triangle DAM cuts lines DB and DC at E and F respectively. Show that $BE = CF$.
3	In a plane we have n lines, no two of which are parallel or perpendicular, and no three of which are concurrent. A cartesian system of coordinates is chosen for the plane with one of the lines as the x-axis. A point P is located at the origin of the coordinate system and starts moving along the positive x-axis with constant velocity. Whenever P reaches the intersection of two lines, it continues along the line it just reached in the direction that increases its x-coordinate. Show that it is possible to choose the system of coordinates in such a way that P visits points from all n lines.
–	Day 2
4	A set X of positive integers is said to be *iberic* if X is a subset of $\{2, 3, \dots, 2018\}$, and whenever m, n are both in X, $\gcd(m, n)$ is also in X. An iberic set is said to be *olympic* if it is not properly contained in any other iberic set. Find all olympic iberic sets that contain the number 33.
5	Let n be a positive integer. For a permutation $a_1, a_2, \dots, a_n$ of the numbers $1, 2, \dots, n$ we define $$b_k = \min_{1 \leq i \leq k} a_i + \max_{1 \leq j \leq k} a_j$$ We say that the permutation $a_1, a_2, \dots, a_n$ is *guadiana* if the sequence $b_1, b_2, \dots, b_n$ does not contain two consecutive equal terms. How many guadiana permutations exist?

6 Let ABC be an acute triangle with $AC > AB > BC$. The perpendicular bisectors of AC and AB cut line BC at D and E respectively. Let P and Q be points on lines AC and AB respectively, both different from A, such that $AB = BP$ and $AC = CQ$, and let K be the intersection of lines EP and DQ. Let M be the midpoint of BC. Show that $\angle DKA = \angle EKM$.