Bangladesh Mathematical Olympiad 2016

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1 BdMO National 2016 Higher Secondary

Problem 1:
(a) Show that $n(n+1)(n+2)$ is divisible by 6.

(b) Show that $1^{2015}+2^{2015}+3^{2015}+4^{2015}+5^{2015}+6^{2015}$ is divisible by 7.

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Problem 2:
(a) How many positive integer factors does 6000 have?

(b) How many positive integer factors of 6000 are not perfect squares?

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3 $\triangle ABC$ is isosceles $AB = AC$. P is a point inside $\triangle ABC$ such that $\angle BCP = 30$ and $\angle APB = 150$ and $\angle CAP = 39$. Find $\angle BAP$.

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Problem 4:
Consider the set of integers $\{1, 2, ..., 100\}$. Let $\{x_1, x_2, ..., x_{100}\}$ be some arbitrary arrangement of the integers $\{1, 2, ..., 100\}$, where all of the x_i are different. Find the smallest possible value of the sum,

$S = |x_2 - x_1| + |x_3 - x_2| + ... + |x_{100} - x_{99}| + |x_1 - x_{100}|$.

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Problem 5:
Suppose there are m Martians and n Earthlings at an intergalactic peace conference. To ensure the Martians stay peaceful at the conference, we must make sure that no two Martians sit together, such that between any two Martians there is always at least one Earthling.

(a) Suppose all $m + n$ Martians and Earthlings are seated in a line. How many ways can the Earthlings and Martians be seated in a line?

(b) Suppose now that the $m + n$ Martians and Earthlings are seated around a circular round-table. How many ways can the Earthlings and Martians be seated around the round-table?

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6 BdMO National 2016 Higher Secondary Problem 6

$\triangle ABC$ is an isosceles triangle with $AC = BC$ and $\angle ACB < 60^\circ$. I and O are the incenter and circumcenter of $\triangle ABC$. The circumcircle of $\triangle BIO$ intersects BC at $D \neq B$.

(a) Do the lines AC and DI intersect? Give a proof.

(b) What is the angle of intersection between the lines OD and IB?

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7 Juli is a mathematician and devised an algorithm to find a husband. The strategy is:
Start interviewing a maximum of 1000 prospective husbands. Assign a ranking r to each person that is a positive integer. No two prospects will have same the rank r.
Reject the first k men and let H be highest rank of these k men.
After rejecting the first k men, select the next prospect with a rank greater than H and then stop the search immediately. If no candidate is selected after 999 interviews, the $1000th$ person is selected.

Juli wants to find the value of k for which she has the highest probability of choosing the highest ranking prospect among all 1000 candidates without having to interview all 1000 prospects.
(a) (6 points:) What is the probability that the highest ranking prospect among all 1000 prospects is the $(m+1)th$ prospect?
(b) (6 points:) Assume the highest ranking prospect is the $(m+1)th$ person to be interviewed. What is the probability that the highest rank candidate among the first m candidates is one of the first k candidates who were rejected?
(c) (6 points:) What is the probability that the prospect with the highest rank is the $(m+1)th$ person and that Juli will choose the $(m+1)th$ man using this algorithm?
(d) (16 points:) The total probability that Juli will choose the highest ranking prospect among the 1000 prospects is the sum of the probability for each possible value of $m+1$ with $m+1$ ranging between $k+1$ and 1000.
Find the sum. To simplify your answer use the formula $InN \approx \frac{1}{N-1} + \frac{1}{N-2} + ... + \frac{1}{2} + 1$
(e) (6 points:) Find that value of k that maximizes the probability of choosing the highest ranking prospect without interviewing all 1000 candidates. You may need to know that the maximum of the function $xln\frac{A}{x-1}$ is approximately $\frac{A+1}{e}$, where A is a constant and e is Eulers number, $e = 2.718....$

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8 Triangle ABC is inscribed in circle ω with $AB = 5$, $BC = 7$, and $AC = 3$. The bisector of angle A meets side BC at D and circle ω at a second point E. Let γ be the circle with diameter DE. Circles ω and γ meet at E and a second point F. Then $AF^2 = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

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9 The integral $Z(0) = \int_{-\infty}^{\infty} dx e^{-x^2} = \sqrt{\pi}$

(a)(3 POINTS:)Show that the integral $Z(j) = \int_{-\infty}^{\infty} dx e^{-x^2+jx}$
Where j is not a function of x,is $Z(j) = e^{j^2/4a} Z(0)$

(b)(10 POINTS):Show that, $\frac{1}{Z(0)} = \int x^{2n}e^{-x^2} = \frac{(2n-1)!!}{2^n}$
Where $(2n-1)!!$ is defined as $(2n-1)(2n-3) \times ... \times 3 \times 1$

(c)(7 POINTS):What is the number of ways to form n pairs from $2n$ distinct objects?Interept the **previous part** of the problem in term of this answer.