

Bangladesh Mathematical Olympiad 2019

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1 Find all prime numbers such that the square of the prime number can be written as the sum of cubes of two positive integers.

2 Prove that, if a, b, c are positive real numbers,

$$\frac{a}{bc} + \frac{b}{ca} + \frac{c}{ab} \geq \frac{2}{a} + \frac{2}{b} - \frac{2}{c}$$

3 Let α and ω be two circles such that ω goes through the center of α . ω intersects α at A and B . Let P any point on the circumference of ω . The lines PA and PB intersect α again at E and F respectively. Prove that $AB = EF$.

4 A is a positive real number. n is positive integer number. Find the set of possible values of the infinite sum $x_0^n + x_1^n + x_2^n + \dots$ where x_0, x_1, x_2, \dots are all positive real numbers so that the infinite series $x_0 + x_1 + x_2 + \dots$ has sum A .

5 Prove that for all positive integers n we can find a permutation of $1, 2, \dots, n$ such that the average of two numbers doesn't appear in-between them. For example $1, 3, 2, 4$ works, but $1, 4, 2, 3$ doesn't because 2 is between 1 and 3 .

6 When a function $f(x)$ is differentiated n times, the function we get is denoted $f^{(n)}(x)$. If $f(x) = \frac{e^x}{x}$. Find the value of

$$\lim_{n \rightarrow \infty} \frac{f^{(2n)}(1)}{(2n)!}$$

7 Given three concentric circles $\omega_1, \omega_2, \omega_3$ with radii r_1, r_2, r_3 such that $r_1 + r_3 \geq 2r_2$. Construct a line that intersects $\omega_1, \omega_2, \omega_3$ at A, B, C respectively such that $AB = BC$.

8 The set of natural numbers \mathbb{N} are partitioned into a finite number of subsets. Prove that there exists a subset S so that for any natural numbers n , there are infinitely many multiples of n in S .

9 Let $ABCD$ is a convex quadrilateral. The internal angle bisectors of $\angle BAC$ and $\angle BDC$ meet at P . $\angle APB = \angle CPD$. Prove that $AB + BD = AC + CD$.

- 10** Given $2020 * 2020$ chessboard, what is the maximum number of warriors you can put on its cells such that no two warriors attack each other.
Warrior is a special chess piece which can move either 3 steps forward and one step sideward and 2 step forward and 2 step sideward in any direction.
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