Bangladesh Mathematical Olympiad 2013

www.artofproblemsolving.com/community/c854534

by Olympus_mountaineer, Ramanujan_1729

1 Higher Secondary P1

A polygon is called degenerate if one of its vertices falls on a line that joins its neighboring two vertices. In a pentagon $ABCDE$, $AB = AE$, $BC = DE$, P and Q are midpoints of AE and AB respectively. $PQ||CD$, BD is perpendicular to both AB and DE. Prove that $ABCDE$ is a degenerate pentagon.

---

2 Higher Secondary P2

Let g be a function from the set of ordered pairs of real numbers to the same set such that $g(x, y) = -g(y, x)$ for all real numbers x and y. Find a real number r such that $g(x, x) = r$ for all real numbers x.

---

3 Higher Secondary P3

Let $ABCDEF$ be a regular hexagon with $AB = 7$. M is the midpoint of DE. AC and BF intersect at P, AC and BM intersect at Q, AM and BF intersect at R. Find the value of $[APB] + [BQC] + [ARF] - [PQMR]$. Here $[X]$ denotes the area of polygon X.

---

4 Higher Secondary P4

If the fraction $\frac{a}{b}$ is greater than $\frac{31}{17}$ in the least amount while $b < 17$, find $\frac{a}{b}$.

---

5 Higher Secondary P5

Let $x > 1$ be an integer such that for any two positive integers a and b, if x divides ab then x either divides a or divides b. Find with proof the number of positive integers that divide x.

---

6 There are n cities in a country. Between any two cities there is at most one road. Suppose that the total number of roads is n. Prove that there is a city such that starting from there it is possible to come back to it without ever travelling the same road twice.

---

7 Higher Secondary P7

If there exists a prime number p such that $p + 2q$ is prime for all positive integer q smaller than p, then p is called an "awesome prime". Find the largest "awesome prime" and prove that it is indeed the largest such prime.

---

8 $\triangle ABC$ is an acute angled triangle. Perpendiculars drawn from its vertices on the opposite sides are AD, BE and CF. The line parallel to DF through E meets BC at Y and BA at X. DF and CA meet at Z. Circumcircle of XYZ meets AC at S. Given, $\angle B = 33^\circ$. find the angle $\angle FSD$ with proof.

---

9 Six points A, B, C, D, E, F are chosen on a circle anticlockwise. None of AB, CD, EF is a diameter. Extended AB and DC meet at Z, CD and FE at X, EF and BA at $Y. AC$ and BF meets at P, CE and BD at Q and AE and DF at R. If O is the point of intersection of YQ and $ZR,$ find the $\angle XOP.$

---

10 Higher Secondary P10

X is a set of n elements. $P_m(X)$ is the set of all m element subsets (i.e. subsets that contain exactly m elements) of X. Suppose $P_m(X)$ has k elements. Prove that the elements of $P_m(X)$ can be ordered in a sequence $A_1, A_2, ...A_i, ...A_k$ such that it satisfies the two conditions:
(A) each element of $P_m(X)$ occurs exactly once in the sequence,
(B) for any i such that $0 < i < k$, the size of the set $A_i \cap A_{i+1}$ is $m - 1$.
