Greece National Olympiad 2019

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1 Define the sequnce $(a_n)_{n\geq 1}$ by $a_1 = 1$ and $a_n = 5a_{n-1} + 3^{n-1}$ for $n \geq 2$.
Find the greatest power of 2 that divides $a_{2^{2019}}$.

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2 Let ABC be a triangle with $AB < AC < BC$.Let O be the center of it's circumcircle and D be the center
of minor arc $\overset{\frown}{AB}$.Line AD intersects BC at E and the circumcircle of BDE intersects AB at Z ,$(Z \neq B)$.The circumcircle of ADZ intersects AC at H ,$(H \neq A)$,prove that $BE = AH$.

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3 Find all positive rational (x, y) that satisfy the equation :

$$yx^y = y + 1$$

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4 Given a $n \times m$ grid we play the following game . Initially we place M tokens in each of M empty cells and at the end of the game we need to fill the whole grid with tokens.For that purpose we are allowed to make the following move:If an empty cell shares a common side with at least two other cells that contain a token then we can place a token in this cell.Find the minimum value of M in terms of m, n that enables us to win the game.
