Greece Team Selection Test 2018

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1 If x, y, z are positive real numbers such that $x+y+z=9xyz$. Prove that:

$$\frac{x}{\sqrt{x^2+2yz+2}}+\frac{y}{\sqrt{y^2+2zx+2}}+\frac{z}{\sqrt{z^2+2xy+2}} \geq 1.$$

Consider when equality applies.

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2 A triangle ABC is inscribed in a circle (C) .Let G the centroid of $\triangle ABC$.
We draw the altitudes AD, BE, CF of the given triangle .Rays AG and GD meet (C) at M and N.Prove that points F, E, M, N are concyclic.

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3 Find all functions $f : \mathbb{Z}_{>0} \mapsto \mathbb{Z}_{>0}$ such that

$$xf(x)+(f(y))^2+2xf(y)$$

is perfect square for all positive integers x, y.

**This problem was proposed by me for the BMO 2017 and it was shortlisted. We then used it in our TST.

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4 Let $p \geq 2$ be a prime number. Eduardo and Fernando play the following game making moves alternately: in each move, the current player chooses an index i in the set $\{0, 1, 2, \ldots, p-1\}$ that was not chosen before by either of the two players and then chooses an element a_i from the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Eduardo has the first move. The game ends after all the indices have been chosen .Then the following number is computed:

$$M = a_0 + a_1 10 + a_2 10^2 + \cdots + a_{p-1} 10^{p-1} = \sum_{i=0}^{p-1} a_i.10^i$$

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The goal of Eduardo is to make M divisible by p, and the goal of Fernando is to prevent this.

Prove that Eduardo has a winning strategy.

Proposed by Amine Natik, Morocco
