

Nordic 2019

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by sqing

- 1 A set of different positive integers is called meaningful if for any finite nonempty subset the corresponding arithmetic and geometric means are both integers. *a)* Does there exist a meaningful set which consists of 2019 numbers? *b)* Does there exist an infinite meaningful set?
Note: The geometric mean of the non-negative numbers a_1, a_2, \dots, a_n is defined as $\sqrt[n]{a_1 a_2 \cdots a_n}$.
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- 2 Let a, b, c be the side lengths of a right angled triangle with c ζ a, b . Show that

$$3 < \frac{c^3 - a^3 - b^3}{c(c-a)(c-b)} \leq \sqrt{2} + 2.$$

- 3 The quadrilateral $ABCD$ satisfies $\angle ACD = 2\angle CAB$, $\angle ACB = 2\angle CAD$ and $CB = CD$. Show that

$$\angle CAB = \angle CAD.$$

- 4 Let n be an integer with $n \geq 3$ and assume that $2n$ vertices of a regular $(4n + 1)$ -gon are coloured. Show that there must exist three of the coloured vertices forming an isosceles triangle.
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