Tuymaada Olympiad 1994

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–	day 1
1	World Cup in America introduced a new point system. For a victory 3 points are given, for a draw 1 point and for defeat 0 points. In the preliminary games, the teams are divided into groups of 4 teams. In groups, teams play with each other, once, then in accordance with the points scored a,b,c and d ($a>b>c>d$) teams take the first, second, third and fourth place in their groups. Give all possible options for the distribution points a,b,c and d
2	The set of numbers $M=\{4k-3 \vert k\in N\}$ is considered. A number of of this set is called simple if it is impossible to put in the form of a product of numbers from M other than 1. Show that in this set, the decomposition of numbers in the product of "simple" factors is ambiguous.
3	Point M lies inside triangle ABC. Prove that for any other point N lying inside the triangle ABC, at least one of the following three inequalities is fulfilled: $AN > AM, BN > BM, CN > CM$.
4	Let a convex polyhedron be given with volume V and full surface S. Prove that inside a polyhedron it is possible to arrange a ball of radius $\frac{V}{S}$.
–	day 2
5	Find the smallest natural number n for which $sin \left(\frac{1}{n+1934}\right) < \frac{1}{1994}$.
6	In three houses A, B and C, forming a right triangle with the legs $AC = 30$ and $CB = 40$, live three beetles a, b and c, capable of moving at speeds of $2, 3$ and 4, respectively. Suppose that you simultaneously release these bugs from point M and mark the time after which beetles reach their homes. Find on the plane such a point M, where is the last time to reach the house a bug would be minimal.
7	Prove that there are infinitely many natural numbers a,b,c,u and v with greatest common divisor 1 satisfying the system of equations: $a+b+c=u+v$ and $a^2+b^2+c^2=u^2+v^2$
8	Prove that in space there is a sphere containing exactly 1994 points with integer coordinates.