Tuymaada Olympiad 1995

www.artofproblemsolving.com/community/c866133

by parmenides51

–	day 1
1	Give a geometric proof of the statement that the fold line on a sheet of paper is straight.
2	Let $x_1 = a, x_2 = a^{x_1}, ..., x_n = a^{x_{n-1}}$ where $a > 1$. What is the maximum value of a for which lim exists $\lim_{n\to\infty} x_n$ and what is this limit?
3	Prove that the equation $(\sqrt{5}+1)^{2x} + (\sqrt{5}-1)^{2x} = 2^x(y^2+2)$ has an infinite number of solutions in natural numbers.
4	It is known that the merchant's n clients live in locations laid along the ring road. Of these, k customers have debts to the merchant for $a_1, a_2, ..., a_k$ rubles, and the merchant owes the remaining $n-k$ clients, whose debts are $b_1, b_2, ..., b_{n-k}$ rubles, moreover, $a_1 + a_2 + ... + a_k = b_1 + b_2 + ... + b_{n-k}$. Prove that a merchant who has no money can pay all his debts and have paid all the customer debts, by starting a customer walk along the road from one of points and not missing any of their customers.
–	day 2
6	Given a circle of radius $r = 1995$. Show that around it you can describe exactly 16 primitive Pythagorean triangles. The primitive Pythagorean triangle is a right-angled triangle, the lengths of the sides of which are expressed by coprime integers.
5	A set consisting of n points of a plane is called an isosceles n-point if any three of its points are located in vertices of an isosceles triangle. Find all natural the numbers for which there exist isosceles n-points.
7	Find a continuous function $f(x)$ satisfying the identity $f(x) - f(ax) = x^n - x^m$, where $n, m \in N, 0 < a < 1$
8	Inside the triangle ABC a point M is given . Find the points P, Q and R lying on the sides AB, BC and AC respectively and such so that the sum $MP + PQ + QR + RM$ is the smallest.