Tuymaada Olympiad 1997

www.artofproblemsolving.com/community/c866135

by parmenides51

–	day 1
1	The product of any three of these four natural numbers is a perfect square. Prove that these numbers themselves are perfect squares.
2	Solve in natural numbers the system of equations $3x^2+6y^2+5z^2=1997$ and $3x+6y+5z=161$.
3	Is it possible to paint all natural numbers in 6 colors, for each one color to be used and the sum of any five numbers of different color to be painted in the sixth color?
4	Using only angle with angle $\frac{\pi}{7}$ and a ruler, constuct angle $\frac{\pi}{14}$
–	day 2
5	Prove the inequality $\left(1+\frac{1}{q}\right)\left(1+\frac{1}{q^2}\right)...\left(1+\frac{1}{q^n}\right)<\frac{q-1}{q-2}$ for $n\in N, q>2$
6	Are there 14 consecutive positive integers, each of which has a divisor other than 1 and not exceeding 11?
7	It is known that every student of the class for Sunday once visited the rink, and every boy met there with every girl. Prove that there was a point in time when all the boys, or all the girls of the class were simultaneously on the rink.
8	Find a right triangle that can be cut into 365 equal triangles.