Tuymaada Olympiad 1998

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–	day 1
1	Write the number $\frac{1997}{1998}$ as a sum of different numbers, inverse to naturals.
2	Solve the equation $(x^3-1000)^{1/2}=(x^2+100)^{1/3}$
3	The segment of length ℓ with the ends on the border of a triangle divides the area of that triangle in half. Prove that $\ell > r\sqrt{2}$, where r is the radius of the inscribed circle of the triangle.
4	Given the tetrahedron $ABCD$, whose opposite edges are equal, that is, $AB=CD, AC=BD$ and $BC=AD$. Prove that exist exactly 6 planes intersecting the triangular angles of the tetrahedron and dividing the total surface and volume of this tetrahedron in half.
–	day 2
5	A right triangle is inscribed in parabola $y=x^2$. Prove that it's hypotenuse is not less than 2.
6	Prove that the sequence of the first digits of the numbers in the form 2^n+3^n is nonperiodic.
7	All possible sequences of numbers -1 and $+1$ of length 100 are considered. For each of them, the square of the sum of the terms is calculated. Find the arithmetic average of the resulting values.
8	Given the pyramid $ABCD$. Let O be the midpoint of edge AC. Given that DO is the height of the pyramid, $AB=BC=2DO$ and the angle ABC is right. Cut this pyramid into 8 equal and similar to it pyramids.