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by a1267ab

1 Let ABC be an acute scalene triangle with incenter I. Show that the circumcircle of BIC intersects the Euler line of ABC in two distinct points.

(Recall that the *Euler line* of a scalene triangle is the line that passes through its circumcenter, centroid, orthocenter, and the nine-point center.)

Andrew Gu

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2 Annie has a permutation $(a_1, a_2, \ldots, a_{2019})$ of $S = \{1, 2, \ldots, 2019\}$, and Yannick wants to guess her permutation. With each guess Yannick gives Annie an n-tuple $(y_1, y_2, \ldots, y_{2019})$ of integers in S, and then Annie gives the number of indices $i \in S$ such that $a_i = y_i$.

(a) Show that Yannick can always guess Annie's permutation with at most 1200000 guesses.
(b) Show that Yannick can always guess Annie's permutation with at most 24000 guesses.

Yannick Yao

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3 Do there exist four points $P_i = (x_i, y_i) \in \mathbb{R}^2$ $(1 \le i \le 4)$ on the plane such that:

- for all $i = 1, 2, 3, 4$, the inequality $x_i^4 + y_i^4 \le x_i^3 + y_i^3$ holds, and
- for all $i \neq j$, the distance between P_i and P_j is greater than 1?

Pakawut Jiradilok

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4 A *cactus* is a finite simple connected graph where no two cycles share an edge. Show that in a nonempty cactus, there must exist a vertex which is part of at most one cycle.

Kevin Yang

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5 Let $p = 2017$ be a prime and $\mathbb{F}_p$ be the integers modulo p. A function $f : \mathbb{Z} \to \mathbb{F}_p$ is called *good* if there is $\alpha \in \mathbb{F}_p$ with $\alpha \not\equiv 0 \pmod{p}$ such that

$$f(x)f(y) = f(x+y) + \alpha^y f(x-y) \pmod{p}$$

for all $x, y \in \mathbb{Z}$. How many good functions are there that are periodic with minimal period 2016?

Ashwin Sah
