Greece JBMO TST 2019

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1 Consider an acute triangle ABC with $AB > AC$ inscribed in a circle of center O. From the midpoint D of side BC we draw line (ℓ) perpendicular to side AB that intersects it at point E. If line AO intersects line (ℓ) at point Z, prove that points A, Z, D, C are concyclic.

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2 Find all pairs of positive integers (x, n) that are solutions of the equation $3 \cdot 2^x + 4 = n^2$.

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3 Let a, b, c be positive real numbers . Prove that

$$\frac{1}{ab(b+1)(c+1)} + \frac{1}{bc(c+1)(a+1)} + \frac{1}{ca(a+1)(b+1)} \geq \frac{3}{(1+abc)^2}.$$

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4 Consider a 8×8 chessboard where all 64 unit squares are at the start white. Prove that, if any 12 of the 64 unit square get painted black, then we can find 4 lines and 4 rows that have all these 12 unit squares.

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