KJMO 2005

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–	day 1
1	Find a irreducible fraction with denominator not greater than 2005, that is closest to $\frac{9}{25}$ but is not $\frac{9}{25}$
2	For triangle ABC, P and Q satisfy $\angle BPA + \angle AQC = 90^o$. It is provided that the vertices of the triangle BAP and ACQ are ordered counterclockwise (or clockwise). Let the intersection of the circumcircles of the two triangles be N ($A \neq N$, however if A is the only intersection $A = N$), and the midpoint of segment BC be M. Show that the length of MN does not depend on P and Q.
3	For a positive integer K, de fine a sequence, $\{a_n\}$, as following: $a_1 = K$ and $a_{n+1} = a_n - 1$ if a_n is even $a_{n+1} = \frac{a_n - 1}{2}$ if a_n is odd , for all $n \geq 1$. Find the smallest value of K, which makes a_{2005} the first term equal to 0.
4	11 students take a test. For any two question in a test, there are at least 6 students who solved exactly one of those two questions. Prove that there are no more than 12 questions in this test. Showing the equality case is not needed.
–	day 2
5	In $\triangle ABC$, let the bisector of $\angle BAC$ hit the circumcircle at M. Let P be the intersection of CM and AB. Denote by (V, WX, YZ) the intersection of the line passing V perpendicular to WX with the line YZ. Prove that the points $(P, AM, AC), (P, AC, AM), (P, BC, MB)$ are collinear. In isosceles triangle APX with $AP = AX$, select a point M on the altitude. PM intersects AX at C. The circumcircle of ACM intersects AP at B. A line passing through P perpendicular to BC intersects MB at Z. Show that XZ is perpendicular to AP.
6	For two different prime numbers p, q, defi ne $S_{p,q} = \{p, q, pq\}$. If two elements in $S_{p,q}$ are numbers in the form of $x^2 + 2005y^2, (x, y \in Z)$, prove that all three elements in $S_{p,q}$ are in such form.
7	If positive reals $x_1, x_2, \cdots, x_n$ satisfy $\sum_{i=1}^n x_i = 1$. Prove that $$\sum_{i=1}^n \frac{1}{1 + \sum_{j=1}^i x_j} < \sqrt{\frac{2}{3} \sum_{i=1}^n \frac{1}{x_i}}$$

8 A group of 6 students decided to make study groups and service activity groups according to the following
principle:
Each group must have exactly 3 members. For any pair of students, there are same number of study groups
and service activity groups that both of the students are members.
Supposing there are at least one group and no three students belong to the same study group and service activity group, prove that the minimum number of groups is 8.