

Benelux 2019

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-Let a, b, c, d be real numbers with $0 \leq a, b, c, d \leq 1$. Prove that

$$ab(a - b) + bc(b - c) + cd(c - d) + da(d - a) \leq \frac{8}{27}.$$

-Find all quadruples (a, b, c, d) of real numbers with $0 \leq a, b, c, d \leq 1$ for which equality holds in the above inequality.

1 Pawns and rooks are placed on a 2019×2019 chessboard, with at most one piece on each of the 2019^2 squares. A rook *can* see another rook if they are in the same row or column and all squares between them are empty. What is the maximal number p for which p pawns and $p + 2019$ rooks can be placed on the chessboard in such a way that no two rooks can see each other?

3 Two circles Γ_1 and Γ_2 intersect at points A and Z (with $A \neq Z$). Let B be the centre of Γ_1 and let C be the centre of Γ_2 . The exterior angle bisector of $\angle BAC$ intersects Γ_1 again at X and Γ_2 again at Y . Prove that the interior angle bisector of $\angle BZC$ passes through the circumcenter of $\triangle XYZ$.

[i]For points P, Q, R that lie on a line ℓ in that order, and a point S not on ℓ , the interior angle bisector of $\angle PQS$ is the line that divides $\angle PQS$ into two equal angles, while the exterior angle bisector of $\angle PQS$ is the line that divides $\angle RQS$ into two equal angles.[/i]

4 An integer $m > 1$ is *rich* if for any positive integer n , there exist positive integers x, y, z such that $n = mx^2 - y^2 - z^2$. An integer $m > 1$ is *poor* if it is not rich.

-Find a poor integer.

-Find a rich integer.
