KJMO 2016

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–	day 1
1	positive reals $a_1, a_2, ...$ satisfying (i) $a_{n+1} = a_1^2 \cdot a_2^2 \cdot ... \cdot a_n^2 - 3$(all positive integers n) (ii) $\frac{1}{2}(a_1 + \sqrt{a_2 - 1})$ is positive integer. prove that $\frac{1}{2}(a_1 \cdot a_2 \cdot ... \cdot a_n + \sqrt{a_{n+1} - 1})$ is positive integer
2	A non-isosceles triangle $\triangle ABC$ has its incircle tangent to BC, CA, AB at points D, E, F. Let the incenter be I. Say AD hits the incircle again at G, at let the tangent to the incircle at G hit AC at H. Let $IH \cap AD = K$, and let the foot of the perpendicular from I to AD be L. Prove that $IE \cdot IK = IC \cdot IL$.
3	n players participated in a competition. Any two players have played exactly one game, and there was no tie game. For a set of $k(\leq n)$ players, if it is able to line the players up so that each player won every player at the back, we call the set *ranked*. For each player who participated in the competition, the set of players who lost to the player is ranked. Prove that the whole set of players can be split into three or less ranked sets.
4	find all positive integer n, satisfying $$\frac{n(n+2016)(n+2\cdot 2016)(n+3\cdot 2016)...(n+2015\cdot 2016)}{1\cdot 2\cdot 3\cdot .....\cdot 2016}$$ is positive integer.
–	day 2
5	$n \in \mathbb{N}^+$ Prove that the following equation can be expressed as a polynomial about n. $$\left[2\sqrt{1}\right] + \left[2\sqrt{2}\right] + \left[2\sqrt{3}\right] + ... + \left[2\sqrt{n^2}\right]$$
6	circle O_1 is tangent to AC, BC(side of triangle ABC) at point D, E. circle O_2 include O_1, is tangent to BC, AB(side of triangle ABC) at point E, F The tangent of O_2 at $P(DE \cap O_2, P \neq E)$ meets AB at Q. A line passing through O_1(center of O_1) and parallel to BO_2(O_2 is also center of O_2) meets BC

at G, $EQ \cap AC = K, KG \cap EF = L$, EO_2 meets circle O_2 at $N(\neq E)$, $LO_2 \cap FN = M$.
IF N is a middle point of FM, prove that $BG = 2EG$

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7 positive integers $a_1, a_2, ..., a_9$ satisfying $a_1 + a_2 + ... + a_9 = 90$
find maximum of

$$\frac{1^{a_1} \cdot 2^{a_2} \cdot ... \cdot 9^{a_9}}{a_1! \cdot a_2! \cdot ... \cdot a_9!}$$

I was really shocked because there are no inequality problems at KJMO
and the test difficulty even more lower...

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8 One moving point in the coordinate plane can move right or up one position. N is a number of all paths : paths that moving point starts from $(0,0)$, without passing $(1,0), (2,1), ..., (n,n-1)$ and moves $2n$ times to (n,n). a_k is a number of special paths : paths include in N, but kth moves to the right, $k+1$th moves to the up.
find

$$\frac{1}{N}(a_1 + a_2 + ... + a_{2n-1})$$
