KJMO 2018

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–	day 1
1	Let f be a quadratic function which satisfies the following condition. Find the value of $\frac{f(8)-f(2)}{f(2)-f(1)}$. For two distinct real numbers a, b, if $f(a) = f(b)$, then $f(a^2 - 6b - 1) = f(b^2 + 8)$.
2	Find all positive integer N which has not less than 4 positive divisors, such that the sum of squares of the 4 smallest positive divisors of N is equal to N.
3	Let there be a scalene triangle ABC, and denote M by the midpoint of BC. The perpendicular bisector of BC meets the circumcircle of ABC at point P, on the same side with A with respect to BC. Let the incenters of ABM and AMC be I, J, respectively. Let $\angle BAC = \alpha$, $\angle ABC = \beta$, $\angle BCA = \gamma$. Find $\angle IPJ$.
4	For a positive integer n, denote $p(n)$ to be the number of nonnegative integer tuples (x, y, z, w) such that $x + 2y + 2z + 3w = n$. Also, denote $q(n)$ to be the number of nonnegative integer tuples (a, b, c, d) such that (i) $a + b + c + d = n$ (ii) $a \geq b \geq d$ (iii) $a \geq c \geq d$ Prove that for all n, $p(n) = q(n)$.
–	day 2
5	Let there be an acute scalene triangle ABC with circumcenter O. Denote D, E be the reflection of O with respect to AB, AC, respectively. The circumcircle of ADE meets AB, AC, the circumcircle of ABC at points K, L, M, respectively, and they are all distinct from A. Prove that the lines BC, KL, AM are concurrent.
6	Let there be a figure with 9 disks and 11 edges, as shown below. We will write a real number in each and every disk. Then, for each edge, we will write the square of the difference between the two real numbers written in the two disks that the edge connects. We must write 0 in disk A, and 1 in disk I. Find the minimum sum of all real numbers written in 11 edges.

7 Find all integer pair (m, n) such that $7^m = 5^n + 24$.

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8 For every set S with $n(\geq 3)$ distinct integers, show that there exists a function $f : \{1, 2, \dots, n\} \rightarrow S$ satisfying the following two conditions.

(i) $\{f(1), f(2), \dots, f(n)\} = S$
(ii) $2f(j) \neq f(i) + f(k)$ for all $1 \leq i < j < k \leq n$.
