ITAMO 2019

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1 Let $ABCDEF$ be a hexagon inscribed in a circle such that $AB = BC$, $CD = DE$ and $EF = AF$. Prove that segments AD, BE and CF are concurrent.

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2 Let p, q be prime numbers. Prove that if $p + q^2$ is a perfect square, then $p^2 + q^n$ is not a perfect square for any positive integer n.

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3 Let $n > 2$ be an integer. We want to color in red exactly $n+1$ of the numbers $1, 2, \cdots, 2n-1, 2n$ so that there do not exists three distinct red integers x, y, z satisfying $x + y = z$. Prove that there is one and one only way to color the red numbers according to the given condition.

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4 Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x.

Let $\lambda \geq 1$ be a real number and n be a positive integer with the property that $\lfloor \lambda^{n+1} \rfloor, \lfloor \lambda^{n+2} \rfloor, \cdots, \lfloor \lambda^{4n} \rfloor$ are all perfect squares. Prove that $\lfloor \lambda \rfloor$ is a perfect square.

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5 Let ABC be an acute angled triangle. Let D be the foot of the internal angle bisector of $\angle BAC$ and let M be the midpoint of AD. Let X be a point on segment BM such that $\angle MXA = \angle DAC$. Prove that AX is perpendicular to XC.

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6 Alberto and Barbara are sitting one next to each other in front of a table onto which they arranged in a line 15 chocolates. Some of them are milk chocolates, while the others are dark chocolates. Starting from Alberto, they play the following game: during their turn, each player eats a positive number of consecutive chocolates, starting from the leftmost of the remaining ones, so that the number of chocolates eaten that are of the same type as the first one is odd (for example, if after some turns the sequence of the remaining chocolates is MMDMD, where M stands for *milk* and D for *dark*, the player could either eat the first chocolate, the first 4 chocolates or all 5 of them). The player eating the last chocolate wins.

Among all 2^{15} possible initial sequences of chocolates, how many of them allow Barbara to have a winning strategy?
