RMM 2018 Shortlist

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– Algebra

A1 Let m and n be integers greater than 2, and let A and B be non-constant polynomials with complex coefficients, at least one of which has a degree greater than 1. Prove that if the degree of the polynomial $A^m - B^n$ is less than $\min(m, n)$, then $A^m = B^n$.

Proposed by Tobi Moektijono, Indonesia

– Combinatorics

C1 Call a point in the Cartesian plane with integer coordinates a $lattice\, point$. Given a finite set $\mathcal{S}$ of lattice points we repeatedly perform the following operation: given two distinct lattice points A, B in $\mathcal{S}$ and two distinct lattice points C, D not in $\mathcal{S}$ such that $ACBD$ is a parallelogram with $AB > CD$, we replace A, B by C, D. Show that only finitely many such operations can be performed.

Proposed by Joe Benton, United Kingdom.

C2 Fix integers $n \geq k \geq 2$. We call a collection of integral valued coins $n - diverse$ if no value occurs in it more than n times. Given such a collection, a number S is $n - reachable$ if that collection contains n coins whose sum of values equals S. Find the least positive integer D such that for any n-diverse collection of D coins there are at least k numbers that are n-reachable.

Proposed by Alexandar Ivanov, Bulgaria.

C3 N teams take part in a league. Every team plays every other team exactly once during the league, and receives 2 points for each win, 1 point for each draw, and 0 points for each loss. At the end of the league, the sequence of total points in descending order $\mathcal{A} = (a_1 \geq a_2 \geq \cdots \geq a_N)$ is known, as well as which team obtained which score. Find the number of sequences $\mathcal{A}$ such that the outcome of all matches is uniquely determined by this information.

Proposed by Dominic Yeo, United Kingdom.

C4 Let k and s be positive integers such that $s < (2k+1)^2$. Initially, one cell out of an $n \times n$ grid is coloured green. On each turn, we pick some green cell c and colour green some s out of the $(2k+1)^2$ cells in the $(2k+1) \times (2k+1)$ square centred at c. No cell may be coloured green twice. We say that s is $k - sparse$ if there exists some positive number C such that, for every positive integer n, the total number of green cells after any number of turns is always going to be at most Cn. Find, in terms of k, the least k-sparse integer s.

Proposed by Nikolai Beluhov.

– Geometry

G1 Let ABC be a triangle and let H be the orthogonal projection of A on the line BC. Let K be a point on the segment AH such that $AH = 3KH$. Let O be the circumcenter of triangle ABC and let M and N be the midpoints of sides AC and AB respectively. The lines KO and MN meet at a point Z and the perpendicular at Z to OK meets lines AB, AC at X and Y respectively. Show that $\angle XKY = \angle CKB$.

Italy

G2 Let $\triangle ABC$ be a triangle, and let S and T be the midpoints of the sides BC and CA, respectively. Suppose M is the midpoint of the segment ST and the circle ω through A, M and T meets the line AB again at N. The tangents of ω at M and N meet at P. Prove that P lies on BC if and only if the triangle ABC is isosceles with apex at A.

Proposed by Reza Kumara, Indonesia

– Number Theory

N1 Determine all polynomials f with integer coefficients such that $f(p)$ is a divisor of $2^p - 2$ for every odd prime p.

Proposed by Italy

N2 Prove that for each positive integer k there exists a number base b along with k triples of Fibonacci numbers (F_u, F_v, F_w) such that when they are written in base b, their concatenation is also a Fibonacci number written in base b. (Fibonacci numbers are defined by $F_1 = F_2 = 1$ and $F_{n+2} = F_{n+1} + F_n$ for all positive integers n.)

Proposed by Serbia