**Peru IMO TST 2013**

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– day 1

**1** Several positive integers are written in a row. Iteratively, Alice chooses two adjacent numbers $x$ and $y$ such that $x > y$ and $x$ is to the left of $y$, and replaces the pair $(x, y)$ by either $(y+1, x)$ or $(x-1, x)$. Prove that she can perform only finitely many such iterations.

*Proposed by Warut Suksompong, Thailand*

**2** Let $a \geq 3$ be a real number, and $P$ a polynomial of degree $n$ and having real coefficients. Prove that at least one of the following numbers is greater than or equal to $1:$

$$|a^0 - P(0)|,\ |a^1 - P(1)|,\ |a^2 - P(2)|, \cdots, |a^{n+1} - P(n+1)|.$$

**3** A point $P$ lies on side $AB$ of a convex quadrilateral $ABCD$. Let $\omega$ be the inscribed circumference of triangle $CPD$ and $I$ the centre of $\omega$. It is known that $\omega$ is tangent to the inscribed circumferences of triangles $APD$ and $BPC$ at points $K$ and $L$ respectively. Let $E$ be the point where the lines $AC$ and $BD$ intersect, and $F$ the point where the lines $AK$ and $BL$ intersect. Prove that the points $E, I, F$ are collinear.

– day 2

**4** Let $A$ be a point outside of a circumference $\omega$. Through $A$, two lines are drawn that intersect $\omega$, the first one cuts $\omega$ at $B$ and $C$, while the other one cuts $\omega$ at $D$ and $E$ ($D$ is between $A$ and $E$). The line that passes through $D$ and is parallel to $BC$ intersects $\omega$ at point $F \neq D$, and the line $AF$ intersects $\omega$ at $T \neq F$. Let $M$ be the intersection point of lines $BC$ and $ET$, $N$ the point symmetrical to $A$ with respect to $M$, and $K$ be the midpoint of $BC$. Prove that the quadrilateral $DEKN$ is cyclic.

**5** Determine all integers $m \geq 2$ such that every $n$ with $\frac{m}{3} \leq n \leq \frac{m}{2}$ divides the binomial coefficient $\binom{n}{m-2n}$.

**6** Players $A$ and $B$ play a game with $N \geq 2012$ coins and $2012$ boxes arranged around a circle. Initially $A$ distributes the coins among the boxes so that there is at least $1$ coin in each box. Then the two of them make moves in the order $B, A, B, A, \ldots$ by the following rules:
**(a)** On every move of his $B$ passes $1$ coin from every box to an adjacent box.
**(b)** On every move of hers $A$ chooses several coins that were *not* involved in $B$'s previous move

and are in different boxes. She passes every coin to an adjacent box.
Player $A$'s goal is to ensure at least $1$ coin in each box after every move of hers, regardless of how $B$ plays and how many moves are made. Find the least $N$ that enables her to succeed.

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