2019 Thailand Mathematical Olympiad

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–	Day 1
1	Let $ABCDE$ be a convex pentagon with $\angle AEB = \angle BDC = 90^o$ and line AC bisects $\angle BAE$ and $\angle DCB$ internally. The circumcircle of ABE intersects line AC again at P. (a) Show that P is the circumcenter of BDE. (b) Show that A, C, D, E are concyclic.
2	Let a, b be two different positive integers. Suppose that a, b are relatively prime. Prove that $\frac{2a(a^2+b^2)}{a^2-b^2}$ is not an integer.
3	Find all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ such that $f(x + yf(x) + y^2) = f(x) + 2y$ for every $x, y \in \mathbb{R}^+$.
4	A rabbit initially stands at the position 0, and repeatedly jumps on the real line. In each jump, the rabbit can jump to any position corresponds to an integer but it cannot stand still. Let $N(a)$ be the number of ways to jump with a total distance of 2019 and stop at the position a. Determine all integers a such that $N(a)$ is odd.
5	Let a, b, c be positive reals such that $abc = 1$. Prove the inequality $$\frac{4a-1}{(2b+1)^2} + \frac{4b-1}{(2c+1)^2} + \frac{4c-1}{(2a+1)^2} \geqslant 1.$$
–	Day 2
6	Determine all function $f : \mathbb{R} \to \mathbb{R}$ such that $xf(y) + yf(x) \leqslant xy$ for all $x, y \in \mathbb{R}$.
7	Let $A = \{-2562, -2561, ..., 2561, 2562\}$. Prove that for any bijection (1-1, onto function) $f : A \to A$, $\sum_{k=1}^{2562} \lvert f(k) - f(-k)\rvert$ is maximized if and only if $f(k)f(-k) < 0$ for any $k = 1, 2, ..., 2562$.
8	Let ABC be a triangle such that $AB \neq AC$ and ω be the circumcircle of this triangle. Let I be the center of the inscribed circle of ABC which touches BC at D.

Let the circle with diameter AI meets ω again at K.
If the line AI intersects ω again at M, show that K, D, M are collinear.

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9 A *chaisri* figure is a triangle which the three vertices are vertices of a regular 2019-gon. Two different chaisri figure may be formed by different regular 2019-gon.
A *thubkaew* figure is a convex polygon which can be dissected into multiple chaisri figure where each vertex of a dissected chaisri figure does not necessarily lie on the border of the convex polygon.

Determine the maximum number of vertices that a thubkaew figure may have.

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10 Prove that there are infinitely many positive odd integer n such that $n!+1$ is composite number.
