National Math Olympiad (Second Round) 2019
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1 We have a rectangle with it sides being a mirror.A light Ray enters from one of the corners of the rectangle and after being reflected several times enters to the opposite corner it started.Prove that at some time the light Ray passed the center of rectangle(Intersection of diagonals.)

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2 ABC is an isosceles triangle ($AB = AC$).
Point X is an arbitrary point on BC. $Z \in AC$ and $Y \in AB$ such that $\angle BXY = \angle ZXC$. A line parallel to YZ passes through B and cuts XZ at T. Prove that AT bisects $\angle A$.

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3 $x_1, x_2, ..., x_n > 1$ are natural numbers and $n \geq 3$
Prove that : $(x_1 x_2 ... x_n)^2 \neq x_1^3 + x_2^3 + ... + x_n^3$

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4 Consider a circle with diameter AB and let C, D be points on its circumcircle such that C, D are not in the same side of AB.Consider the parallel line to AC passing from D and let it intersect AB at E.Similarly consider the paralell line to AD passing from C and let it intersect AB at F.The perpendicular line to AB at E intersects BC at X and the perpendicular line to AB at F intersects DB at Y.Prove that the permiter of triangle AXY is twice CD.

Remark: This problem is proved to be wrong due to a typo in the exam papers you can find the correct version here (`https://artofproblemsolving.com/community/c6h1832731_geometry__iran_mo_2019`).

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5 Ali and Naqi are playing a game. At first, they have Polynomial $P(x) = 1 + x^{1398}$.
Naqi starts. In each turn one can choice natural number $k \in [0, 1398]$ in his trun, and add x^k to the polynomial. For example after 2 moves P can be : $P(x) = x^{1398} + x^{300} + x^{100} + 1$. If after Ali's turn, there exist $t \in R$ such that $P(t) < 0$ then Ali loses the game. Prove that Ali can play forever somehow he never loses the game!

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6 Consider lattice points of a $6 * 7$ grid.We start with two points A, B.We say two points X, Y connected if one can reflect several times WRT points A, B and reach from X to Y.Over all choices of A, B what is the minimum number of connected components?
