Turkey Junior National Olympiad 2018

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by electrovector

1   Let $s(n)$ be the number of positive integer divisors of n.
Find the all positive values of k that is providing $k = s(a) = s(b) = s(2a + 3b)$.

2   We are placing rooks on a $n \cdot n$ chess table that providing this condition:
Every two rooks will threaten an empty square at least.
What is the most number of rooks?

3   In an acute ABC triangle which has a circumcircle center called O, there is a line that perpen-
diculars to AO line cuts $[AB]$ and $[AC]$ respectively on D and E points. There is a point called
K that is different from AO and BC's junction point on $[BC]$. AK line cuts the circumcircle of
ADE on L that is different from A. M is the symmetry point of A according to DE line. Prove
that K,L,M,O are circular.

4   For all x, y, z positive real numbers, find the all c positive real numbers that providing

$$\frac{x^3y + y^3z + z^3x}{x + y + z} + \frac{4c}{xyz} \geq 2c + 2$$