Regional Competition For Advanced Students 2018

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1 If a, b are positive reals such that $a+b<2$. Prove that

$$\frac{1}{1+a^2}+\frac{1}{1+b^2}\leq\frac{2}{1+ab}$$

and determine all a, b yielding equality.

Proposed by Gottfried Perz

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2 Let k be a circle with radius r and AB a chord of k such that $AB>r$. Furthermore, let S be the point on the chord AB satisfying $AS=r$. The perpendicular bisector of BS intersects k in the points C and D. The line through D and S intersects k for a second time in point E. Show that the triangle CSE is equilateral.

Proposed by Stefan Leopoldseder

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3 Let $n\geq 3$ be a natural number.
Determine the number a_n of all subsets of $\{1, 2, ..., n\}$ consisting of three elements such that one of them is the arithmetic mean of the other two.

Proposed by Walther Janous

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4 Let $d(n)$ be the number of all positive divisors of a natural number $n\geq 2$.
Determine all natural numbers $n\geq 3$ such that $d(n-1)+d(n)+d(n+1)\leq 8$.

Proposed by Richard Henner
