Regional Competition For Advanced Students 2010

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by FelixD

1 Let $0 \leq a, b \leq 1$ be real numbers. Prove the following inequality:

$$\sqrt{a^3b^3} + \sqrt{(1-a^2)(1-ab)(1-b^2)} \leq 1.$$

(41th Austrian Mathematical Olympiad, regional competition, problem 1)

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2 Solve the following in equation in $\mathbb{R}^3$:

$$4x^4 - x^2(4y^4 + 4z^4 - 1) - 2xyz + y^8 + 2y^4z^4 + y^2z^2 + z^8 = 0.$$

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3 Let $\triangle ABC$ be a triangle and let D be a point on side $\overline{BC}$. Let U and V be the circumcenters of triangles $\triangle ABD$ and $\triangle ADC$, respectively. Show, that $\triangle ABC$ and $\triangle AUV$ are similar.

(41th Austrian Mathematical Olympiad, regional competition, problem 3)

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4 Let $(b_n)_{n\geq 0} = \sum_{k=0}^{n}(a_0 + kd)$ for positive integers a_0 and d. We consider all such sequences containing an element b_i which equals 2010. Determine the greatest possible value of i and for this value the integers a_0 and d.

(41th Austrian Mathematical Olympiad, regional competition, problem 4)
