Centroamerican and Caribbean Math Olympiad 2019

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– Day 1

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1 Let $N = \overline{abcd}$ be a positive integer with four digits. We name *plátano power* to the smallest positive integer $p(N) = \overline{\alpha_1\alpha_2\ldots\alpha_k}$ that can be inserted between the numbers $\overline{ab}$ and $\overline{cd}$ in such a way the new number $\overline{ab\alpha_1\alpha_2\ldots\alpha_k cd}$ is divisible by N. Determine the value of $p(2025)$.

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2 We have a regular polygon P with 2019 vertices, and in each vertex there is a coin. Two players *Azul* and *Rojo* take turns alternately, beginning with Azul, in the following way: first, Azul chooses a triangle with vertices in P and colors its interior with blue, then Rojo selects a triangle with vertices in P and colors its interior with red, so that the triangles formed in each move don't intersect internally the previous colored triangles. They continue playing until it's not possible to choose another triangle to be colored. Then, a player wins the coin of a vertex if he colored the greater quantity of triangles incident to that vertex (if the quantities of triangles colored with blue or red incident to the vertex are the same, then no one wins that coin and the coin is deleted). The player with the greater quantity of coins wins the game. Find a winning strategy for one of the players.

Note. Two triangles can share vertices or sides.

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3 Let ABC be a triangle and Γ its circumcircle. Let D be the foot of the altitude from A to the side BC, M and N the midpoints of AB and AC, and Q the point on Γ diametrically opposite to A. Let E be the midpoint of DQ. Show that the lines perpendicular to EM and EN passing through M and N, respectively, meet on AD.

– Day 2

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4 Let ABC be a triangle, Γ its circumcircle and l the tangent to Γ through A. The altitudes from B and C are extended and meet l at D and E, respectively. The lines DC and EB meet Γ again at P and Q, respectively. Show that the triangle APQ is isosceles.

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5 Let $a,\ b$ and c be positive real numbers so that $a+b+c=1$. Show that

$$a\sqrt{a^2+6bc}+b\sqrt{b^2+6ac}+c\sqrt{c^2+6ab}\leq\frac{3\sqrt{2}}{4}$$

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6 A *triminó* is a rectangular tile of 1×3. Is it possible to cover a 8×8 chessboard using 21 triminós, in such a way there remains exactly one 1×1 square without covering? In case the answer is in the affirmative, determine all the possible locations of such a unit square in the chessboard.