All Soviet Union Mathematical Olympiad 1972

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by parmenides51

159 Given a rectangle $ABCD$, points M – the middle of $[AD]$ side, N – the middle of $[BC]$ side. Let us take a point P on the extension of the $[DC]$ segment over the point D. Let us denote the intersection point of lines (PM) and (AC) as Q. Prove that the angles QNM and MNP are equal.

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160 Given 50 segments on the line. Prove that one of the following statements is valid:
1. Some 8 segments have the common point.
2. Some 8 segments do not intersect each other.

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161 Find the maximal x such that the expression $4^{27} + 4^{1000} + 4^x$ is the exact square.

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162 a) Let a, n, m be natural numbers, $a > 1$.
Prove that if $(a^m + 1)$ is divisible by $(a^n + 1)$ than m is divisible by n.
b) Let a, b, n, m be natural numbers, $a > 1, a$ and b are relatively prime.
Prove that if $(a^m + b^m)$ is divisible by $(a^n + b^n)$ than m is divisible by n.

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163 The triangle table is built according to the rule: You put the natural number $a > 1$ in the upper row, and then you write under the number k from the left side k^2, and from the right side – $(k + 1)$. For example, if $a = 2$, you get the table on the picture. Prove that all the numbers on each particular line are different.

2
/
leavevmode
/
leavevmode
4 3
/ /
leavevmode
16 5 9 4
/ / / /

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164 Given several squares with the total area 1.
Prove that you can pose them in the square of the area 2 without any intersections.

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165 Let O be the intersection point of the of the convex quadrangle $ABCD$ diagonals.
Prove that the line drawn through the points of intersection of the medians of AOB and COD

triangles is orthogonal to the line drawn through the points of intersection of the heights of BOC and AOD triangles.

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166 Each of the 9 straight lines divides the given square onto two quadrangles with the areas related as $2:3$.
Prove that there exist three of them intersecting in one point

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167 The 7-gon $A_1A_2A_3A_4A_5A_6A_7$ is inscribed in a circle.
Prove that if the centre of the circle is inside the 7-gon , than the sum of A_1, A_2 and A_3 angles is less than 450 degrees.

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168 A game for two.
One gives a digit and the second substitutes it instead of a star in the following difference: $****-****=$
Then the first gives the next digit, and so on 8 times.
The first wants to obtain the greatest possible difference, the second – the least. Prove that:
1. The first can operate in such a way that the difference would be not less than 4000, not depending on the second's behaviour.
2. The second can operate in such a way that the difference would be not greater than 4000, not depending on the first's behaviour.

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169 Let x, y be positive numbers, s – the least of $\{x, (y+1/x), 1/y\}$.
What is the greatest possible value of s? To what x and y does it correspond?

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170 The point O inside the convex polygon makes isosceles triangle with all the pairs of its vertices.
Prove that O is the centre of the circumscribed circle.

other formulation:

P is a convex polygon and X is an interior point such that for every pair of vertices A, B, the triangle XAB is isosceles. Prove that all the vertices of P lie on a circle with center X.

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171 Is it possible to put the numbers $0, 1$ or 2 in the unit squares of the cross-lined paper $100x100$ in such a way, that every rectangle $3x4$ (and $4x3$) would contain three zeros, four ones and five twos?

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172 Let the sum of positive numbers $x_1, x_2, ..., x_n$ be 1.
Let s be the greatest of the numbers $\left\{\frac{x_1}{1+x_1}, \frac{x_2}{1+x_1+x_2}, ..., \frac{x_n}{1+x_1+...+x_n}\right\}$.
What is the minimal possible s? What $x_i$correspond it?

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173 One-round hockey tournament is finished (each plays with each one time, the winner gets 2 points, looser – 0, and 1 point for draw). For arbitrary subgroup of teams there exists a team (may be from that subgroup) that has got an odd number of points in the games with the teams of the subgroup. Prove that there was even number of the participants.