All Soviet Union Mathematical Olympiad 1973

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174 Fourteen coins are submitted to the judge. An expert knows, that the coins from number one to seven are false, and from 8 to 14 – normal. The judge is sure only that all the true coins have the same weight and all the false coins weights equal each other, but are less then the weight of the true coins. The expert has the scales without weights.
a) The expert wants to prove, that the coins $1--7$ are false. How can he do it in three weighings?
b) How can he prove, that the coins $1--7$ are false and the coins $8--14$ are true in three weighings?

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175 Prove that 9-digit number, that contains all the decimal digits except zero and does not ends with 5 can not be exact square.

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176 Given n points, $n > 4$. Prove that tou can connect them with arrows, in such a way, that you can reach every point from every other point, having passed through one or two arrows. (You can connect every pair with one arrow only, and move along the arrow in one direction only.)

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177 Given an angle with the vertex O and a circle touching its sides in the points A and B. A ray is drawn from the point A parallel to $[OB)$. It intersects with the circumference in the point C. The segment $[OC]$ intersects the circumference in the point E. The straight lines (AE) and (OB) intersect in the point K. Prove that $|OK| = |KB|$.

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178 The real numbers a, b, c satisfy the condition:
for all x, such that for $-1 \le x \le 1$, the inequality$|ax^2 + bx + c| \le 1$ is held.
Prove that for the same x , $|cx^2 + bx + a| \le 2$.

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179 The tennis federation has assigned numbers to 1024 sportsmen, participating in the tournament, according to their skill. (The tennis federation uses the olympic system of tournaments. The looser in the pair leaves, the winner meets with the winner of another pair. Thus, in the second tour remains 512 participants, in the third – 256, et.c. The winner is determined after the tenth tour.) It comes out, that in the play between the sportsmen whose numbers differ more than on 2 always win that whose number is less.
What is the greatest possible number of the winner?

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180 The square polynomial $f(x) = ax^2 + bx + c$ is of such a sort, that the equation $f(x) = x$ does not have real roots. Prove that the equation $f(f(x)) = 0$ does not have real roots also.

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181 n squares of the infinite cross-lined sheet of paper are painted with black colour (others are white). Every move all the squares of the sheet change their colour simultaneously. The square

gets the colour, that had the majority of three ones: the square itself, its neighbour from the right side and its neighbour from the upper side.
a) Prove that after the finite number of the moves all the black squares will disappear.
b) Prove that it will happen not later than on the n-th move

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182 Three similar acute-angled triangles AC_1B, BA_1C and CB_1A are built on the outer side of the acute-angled triangle ABC. (Equal triples of the angles are AB_1C, ABC_1, A_1BC and BA_1C, BAC_1, B_1AC.)
a) Prove that the circles circumscribed around the outer triangles intersect in one point.
b) Prove that the straight lines AA_1, BB_1 and CC_1 intersect in the same point

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183 N men are not acquainted each other. You need to introduce some of them to some of them in such a way, that no three men will have equal number of acquaintances. Prove that it is possible for all N.

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184 The king have revised the chess-board 8×8 having visited all the fields once only and returned to the starting point. When his trajectory was drawn (the centres of the squares were connected with the straight lines), a closed broken line without self-intersections appeared.
a) Give an example that the king could make 28 steps parallel the sides of the board only.
b) Prove that he could not make less than 28 such a steps.
c) What is the maximal and minimal length of the broken line if the side of a field is 1?

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185 Given a triangle with a, b, c sides and with the area 1 ($a \geq b \geq c$). Prove that $b^2 \geq 2$.

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186 Given a convex n-angle with pairwise (mutually) non-parallel sides and a point inside it. Prove that there are not more than n straight lines coming through that point and halving the area of the n-angle.

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187 Prove that for every positive x_1, x_2, x_3, x_4, x_5 holds inequality: $(x_1+x_2+x_3+x_4+x_5)^2 \geq 4(x_1x_2+x_3x_4+x_5x_1+x_2x_3+x_4x_5)$.

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188 Given 4 points in three-dimensional space, not lying in one plane.
What is the number of such a parallelepipeds (bricks), that each point is a vertex of each parallelepiped?
