All Soviet Union Mathematical Olympiad 1974

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189 Given some cards with either ”-1” or ”$+1$” written on the opposite side. You are allowed to choose a triple of cards and ask about the product of the three numbers on the cards.
What is the minimal number of questions allowing to determine all the numbers on the cards ...
a) for 30 cards,
b) for 31 cards,
c) for 32 cards.
(You should prove, that you cannot manage with less questions.)
d) Fifty above mentioned cards are lying along the circumference. You are allowed to ask about the product of three consecutive numbers only. You need to determine the product af all the 50 numbers.
What is the minimal number of questions allowing to determine it?

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190 Among all the numbers representable as $36^k - 5^l$ (k and l are natural numbers) find the smallest. Prove that it is really the smallest.

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191 a) Each of the side of the convex hexagon is longer than 1. Does it necessary have a diagonal longer than 2?
b) Each of the main diagonals of the convex hexagon is longer than 2. Does it necessary have a side longer than 1?

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192 Given two circles with the radiuses R and r, touching each other from the outer side. Consider all the trapezoids, such that its lateral sides touch both circles, and its bases touch different circles. Find the shortest possible lateral side.

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193 Given n vectors of unit length in the plane. The length of their total sum is less than one.
Prove that you can rearrange them to provide the property:
for every $k, k \le n$, the length of the sum of the first k vectors is less than 2.

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194 Find all the real a, b, c such that the equality $|ax+by+cz|+|bx+cy+az|+|cx+ay+bz| = |x|+|y|+|z|$ is valid for all the real x, y, z.

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195 Given a square $ABCD$. Points P and Q are in the sides $[AB]$ and $[BC]$ respectively. $|BP| = |BQ|$. Let H be the foot of the perpendicular from the point B to the segment $[PC]$. Prove that the angle DHQ is a right one.

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196 Given some red and blue points. Some of them are connected by the segments. Let us call ”exclusive” the point, if its colour differs from the colour of more than half of the connected points. Every move one arbitrary ”exclusive” point is repainted to the other colour. Prove that after the finite number of moves there will remain no ”exclusive” points.

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197 Find all the natural n and k such that n^n has k digits and k^k has n digits.

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198 Given points D and E on the legs $[CA]$ and $[CB]$, respectively, of the isosceles right triangle. $|CD| = |CE|$. The extensions of the perpendiculars from D and C to the line AE cross the hypotenuse AB in the points K and L. Prove that $|KL| = |LB|$

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199 Two are playing the game ”cats and rats” on the chess-board 8×8. The first has one piece – a rat, the second – several pieces – cats. All the pieces have four available moves – up, down, left, right – to the neighbour field, but the rat can also escape from the board if it is on the boarder of the chess-board. If they appear on the same field – the rat is eaten. The players move in turn, but the second can move all the cats in independent directions.
a) Let there be two cats. The rat is on the interior field. Is it possible to put the cats on such a fields on the border that they will be able to catch the rat?
b) Let there be three cats, but the rat moves twice during the first turn. Prove that the rat can escape.

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200 a) Prove that you can rearrange the numbers $1, 2, ..., 32$ in such a way, that for every couple of numbers none of the numbers between them will equal their arithmetic mean.
b) Can you rearrange the numbers $1, 2, ..., 100$ in such a way, that for every couple of numbers none of the numbers between them will equal their arithmetic mean?

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201 Find all the three-digit numbers such that it equals to the arithmetic mean of the six numbers obtained by rearranging its digits.

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202 Given a convex polygon. You can put no triangle with area 1 inside it.
Prove that you can put the polygon inside a triangle with the area 4.

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203 Given a function $f(x)$ on the segment $0 \le x \le 1$. For all $x, f(x) \ge 0, f(1) = 1$.
For all the couples of (x_1, x_2) such, that all the arguments are in the segment $f(x_1 + x_2) \ge f(x_1) + f(x_2)$.
a) Prove that for all x holds $f(x) \le 2x$.
b) Is the inequality $f(x) \le 1.9x$ valid?

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204 Given a triangle ABC with the are 1. Let A', B' and C' are the middles of the sides $[BC]$, $[CA]$ and $[AB]$ respectively. What is the minimal possible area of the common part of two triangles $A'B'C'$ and KLM, if the points K, L and M are lying on the segments $[AB']$, $[CA']$ and $[BC']$ respectively?
