**All Soviet Union Mathematical Olympiad 1975**

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**205** a) The triangle $ABC$ was turned around the centre of the circumscribed circle by the angle less than $180$ degrees and thus was obtained the triangle $A_1B_1C_1$. The corresponding segments $[AB]$ and $[A_1B_1]$ intersect in the point $C_2, [BC]$ and $[B_1C_1]$ – $A_2, [AC]$ and $[A_1C_1]$ – $B_2$. Prove that the triangle $A_2B_2C_2$ is similar to the triangle $ABC$.
b) The quadrangle $ABCD$ was turned around the centre of the circumscribed circle by the angle less than $180$ degrees and thus was obtained the quadrangle $A_1B1C_1D_1$. Prove that the points of intersection of the corresponding lines ( $(AB)$ and $(A_1B_1), (BC)$ and $(B_1C_1), (CD)$ and $(C_1D_1), (DA)$ and $(D_1A_1)$ ) are the vertices of the parallelogram.

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**206** Given a triangle $ABC$ with the unit area. The first player chooses a point $X$ on the side $[AB]$, than the second – $Y$ on $[BC]$ side, and, finally, the first chooses a point $Z$ on $[AC]$ side. The first tries to obtain the greatest possible area of the $XYZ$ triangle, the second – the smallest. What area can obtain the first for sure and how?

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**207** What is the smallest perimeter of the convex $32$-angle, having all the vertices in the nodes of cross-lined paper with the sides of its squares equal to $1$?

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**208** a) Given a big square consisting of $7 \times 7$ squares. You should mark the centres of $k$ points in such a way, that no quadruple of the marked points will be the vertices of a rectangle with the sides parallel to the sides of the given squares. What is the greatest $k$ such that the problem has solution?
b) The same problem for $13 \times 13$ square.

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**209** Denote the middles of the convex hexagon $A_1A_2A_3A_4A_5A_6$ diagonals $A_6A_2, A_1A_3, A_2A_4, A_3A_5, A_4A_6, A_5A_1$ as $B_1, B_2, B_3, B_4, B_5, B_6$ respectively. Prove that if the hexagon $B_1B_2B_3B_4B_5B_6$ is convex, than its area equals to the quarter of the initial hexagon.

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**210** Prove that it is possible to find $2^{n+1}$ of $2^n$ digit numbers containing only "$1$" and "$2$" as digits, such that every two of them distinguish at least in $2^{n-1}$ digits.

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**211** Given a finite set of polygons in the plane. Every two of them have a common point.
Prove that there exists a straight line, that crosses all the polygons.

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**212** Prove that for all the positive numbers $a, b, c$ the following inequality is valid: $a^3+b^3+c^3+3abc > ab(a+b)+bc(b+c)+ac(a+c)$

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**213** Three flies are crawling along the perimeter of the $ABC$ triangle in such a way, that the centre of their masses is a constant point. One of the flies has already passed along all the perimeter. Prove that the centre of the flies' masses coincides with the centre of masses of the $ABC$ triangle. (The centre of masses for the triangle is the point of medians intersection.

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**214** Several zeros, ones and twos are written on the blackboard. An anonymous clean in turn pairs of different numbers, writing, instead of cleaned, the number not equal to each. ($0$ instead of pair $\{1,2\}, 1$ instead of $\{0,2\}, 2$ instead of $\{0,1\}$). Prove that if there remains one number only, it does not depend on the processing order.

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**215** Given a horizontal strip on the plane (its sides are parallel lines) and $n$ lines intersecting the strip. Every two of them intersect inside the strip, and not a triple has a common point. Consider all the paths along the segments of those lines, starting on the lower side of the strip and ending on the upper side with the properties: moving along such a path we are constantly rising up, and, having reached the intersection, we are obliged to turn to another line. Prove that:
a) there are not less than $n/2$ such a paths without common points;
b) there is a path consisting of not less than of $n$ segments;
c) there is a path that goes along not more than along $n/2+1$ lines;
d) there is a path that goes along all the $n$ lines.

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**216** For what $k$ is it possible to construct a cube $kxkxk$ of the black and white cubes $1x1x1$ in such a way that every small cube has the same colour, that have exactly two his neighbours. (Two cubes are neighbours, if they have the common face.)

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**217** Given a polynomial $P(x)$ with
a) natural coefficients;
b) integer coefficients;
Let us denote with $a_n$ the sum of the digits of $P(n)$ value.
Prove that there is a number encountered in the sequence $a_1, a_2, ..., a_n, ...$ infinite times.

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**218** The world and the european champion are determined in the same tournament carried in one round. There are $20$ teams and $k$ of them are european. The european champion is determined according to the results of the games only between those $k$ teams.
What is the greatest $k$ such that the situation, when the single european champion is the single world outsider, is possible if:
a) it is hockey (draws allowed)?
b) it is volleyball (no draws)?

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**219** a) Given real numbers $a_1, a_2, b_1, b_2$ and positive $p_1, p_2, q_1, q_2$. Prove that in the table $2\times 2$ $(a_1 + b_1)/(p_1 + q_1)$ ,$(a_1 + b_2)/(p_1 + q_2)$ $(a_2 + b_1)/(p_2 + q_1)$ ,$(a_2 + b_2)/(p_2 + q_2)$

there is a number in the table, that is not less than another number in the same row and is not greater than another number in the same column (a saddle point).

b) Given real numbers $a_1, a_2, ..., a_n, b_1, b_2, ..., b_n$ and positive $p_1, p_2, ..., p_n, q_1, q_2, ..., q_n$. We build the table $n \times n$, with the numbers ($0 < i, j \le n$)

$(a_i + b_j)/(p_i + q_j)$

in the intersection of the $i$-th row and $j$-th column.
Prove that there is a number in the table, that is not less than arbitrary number in the same row and is not greater than arbitrary number in the same column (a saddle point).

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