Bosnia and Herzegovina Junior BMO TST 2019

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1 Let x,y,z be real numbers ($x \neq y$, $y \neq z$, $x \neq z$) different from 0. If $\frac{x^2-yz}{x(1-yz)} = \frac{y^2-xz}{y(1-xz)}$, prove that the following relation holds:

$$x+y+z=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}.$$

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2 $2.$ Let ABC be a triangle and AD the angle bisector ($D \in BC$). The perpendicular from B to AD cuts the circumcircle of triangle ABD at E. If O is the center of the circle around ABC , prove A, O, E are collinear.

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3 $3.$ Let S be the set of all positive integers from 1 to 100 included. Two players play a game. The first player removes any k numbers he wants, from S. The second player's goal is to pick k different numbers, such that their sum is 100. Which player has the winning strategy if : $a)$ $k=9$? $b)$ $k=8$?

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4 $4.$ Let there be a variable positive integer whose last two digits are $3's$. Prove that this number is divisible by a prime greater than 7.
