Serbia JBMO TST 2019

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1 Does there exist a positive integer n, such that the number of divisors of $n!$ is divisible by 2019?

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2 Let $a, b, c \in (0, 1)$. Prove that

$$a + b + c + 2abc > ab + bc + ca + 2\sqrt{abc}.$$

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3 3. Congruent circles k_1 and k_2 intersect in the points A and B. Let P be a variable point of arc AB of circle k_2 which is inside k_1 and let AP intersect k_1 once more in point C, and the ray CB intersects k_2 once more in D. Let the angle bisector of $\angle CAD$ intersect k_1 in E, and the circle k_2 in F. Ray FB intersects k_1 in Q. If X is one of the intersection points of circumscribed circles of triangles CDP and EQF, prove that the triangle CFX is equilateral.

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4 4. On a table there are notes of values: 1, 2, 5, 10, 20 ,50, 100, 200, 500, 1000, 2000 and 5000 (the number of any of these notes can be any non-negative integer). Two players , First and Second play a game in turns (First plays first). With one move a player can take any one note of value higher than 1 , and replace it with notes of less value. The value of the chosen note is equal to the sum of the values of the replaced notes. The loser is the player which can not play any more moves. Which player has the winning strategy?
