JHMMC 8 2003

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by JustKeepRunning

1	Jane has 4 pears, 5 bananas, 3 lemons, 1 orange, and 6 apples. If she uses one of each fruit to make a fruit smoothie, what is the total number of fruits that she has left?
2	Philip has 3 triangles and 6 pentagons. Let S be the total number of sides of the shapes he has. Let N be the number of shapes he has. What is $S + N$?
3	On an exam with 80 problems, Roger solved 68 of them. What percentage of the problems did he solve?
4	A number plus 4 is 2003. What is the number?
5	In $\triangle ABC$, $BC = 4$ and $CA = 6$. If the perimeter of the triangle is 4 times the length of side BC, what is the length of AB?
6	Compute $\frac{55}{21} \times \frac{28}{5} \times \frac{3}{2}$.
7	Yao Ming is 7 ft and 5 in tall. His basketball hoop is 10 feet from the ground. Given that there are 12 inches in a foot, how many inches must Yao jump to touch the hoop with his head?
8	What is the area of a square in square feet, if each of its diagonals is 4 feet long?
9	Compute the product of the integers from -5 to 5, inclusive.
10	Let A be the sum of seven 7s. Let B be the sum of seven As. What is B?
11	If a certain number is doubled and the result is increased by 11, the final number is 23. What is the original number?
12	Compute $\frac{664.02}{9.3}$.
13	A problem author for a math competition was looking through a tentative exam when he realized that he could not use one of his proposed problems. Frustrated, he decided to take a nap instead, and slept from $10:47$ AM to $7:32$ PM. For how many minutes did he sleep?

14	In rectangle $ABCD$, $AB = 7$ and $AC = 25$. What is its area?
15	Evaluate $\frac{100-99+98-97\cdots+4-3+2-1}{1-2+3-4\cdots+97-98+99-100}$.
16	A lazy student used the approximation $\pi = \frac{22}{7}$ to calculate the circumference of a given circle. If his answer was 6, what was the radius of the circle?
17	Find the largest divisor of 2800 that is a perfect square.
18	How many multiples of 17 are there between 23 and 227?
19	Two angles are supplementary, and one angle is 9 times as large as the other. What is the number of degrees in the measure of the larger angle?
20	How many positive whole numbers less than 100 are divisible by 3, but not by 2?
21	The surface area and the volume of a cube are numerically equal. Find the cubes volume.
22	Given that $\|3 - a\| = 2$, compute the sum of all possible values of a.
23	Let $ABCD$ be a square with side length 8. A second square $A_1B_1C_1D_1$ is formed by joining the midpoints of AB, BC, CD and DA. A third square $A_2B_2C_2D_2$ is formed in the same way from $A_1B_1C_1D_1$, and a fourth square $A_3B_3C_3D_3$ from $A_2B_2C_2D_2$. Find the sum of the areas of these four squares.
24	If $a + b = 13, b + c = 14, c + a = 15$, find the value of c.
25	Two positive whole numbers differ by 3. The sum of their squares is 117. Find the larger of the two numbers.
26	Given that $5^3 + 5^3 + 5^3 + 5^3 + 5^3 = 5^J$ and $3^2 + 3^2 + 3^2 = 3^N$, what is the value of J^N ?
27	A pair of positive integers a and b is such that their greatest common divisor is 5 and their least common multiple is 55. Find the smallest possible value of $a + b$.
28	How many of the positive divisors of 120 are divisible by 4?

29 How many three-digit numbers are perfect squares?

30 Calculate $1 + 3 + 5 + \cdots + 195 + 197 + 199$

31 The ages of Mr. and Mrs. Fibonacci are both two-digit numbers. If Mr. Fibonaccis age can be formed
by reversing the digits of Mrs. Fibonaccis age, find the smallest possible positive difference between
their ages.

32 Let N be the product of the first nine multiples of 19 (i.e. $N = 19 \times 38 \times 57 \times \cdots \times 152 \times 171$). What is the last digit of N?