All Soviet Union Mathematical Olympiad 1980

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284 All the two-digit numbers from 19 to 80 are written in a line without spaces.
Is the obtained number $192021....7980$ divisible by 1980?

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285 The vertical side of a square is divided onto n segments. The sum of the segments with even numbers lengths equals to the sum of the segments with odd numbers lengths. $n-1$ lines parallel to the horizontal sides are drawn from the segments ends, and, thus, n strips are obtained. The diagonal is drawn from the lower left corner to the upper right one. This diagonal divides every strip onto left and right parts.
Prove that the sum of the left parts of odd strips areas equals to the sum of the right parts of even strips areas.

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286 The load for the space station ”Salute” is packed in containers. There are more than 35 containers, and the total weight is 18 metric tons. There are 7 one-way transport spaceships ”Progress”, each able to bring 3 metric tons to the station. It is known that they are able to take an arbitrary subset of 35 containers. Prove that they are able to take all the load.

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287 The points M and P are the middles of $[BC]$ and $[CD]$ sides of a convex quadrangle $ABCD$.
It is known that $|AM| + |AP| = a$. Prove that the $ABCD$ area is less than $\frac{a^2}{2}$.

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288 Are there three integers x, y, z, such that $x^2 + y^3 = z^4$?

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289 Given a point E on the diameter AC of the certain circle.
Draw a chord BD to maximise the area of the quadrangle $ABCD$.

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290 There are several settlements on the bank of the Big Round Lake. Some of them are connected with the regular direct ship lines. Two settlements are connected if and only if two next (counterclockwise) to each ones are not connected.
Prove that You can move from the arbitrary settlement to another arbitrary settlement, having used not more than three ships.

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291 The six-digit decimal number contains six different non-zero digits and is divisible by 37.
Prove that having transposed its digits you can obtain at least 23 more numbers divisible by 37

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292 Find real solutions of the system :

$$\begin{cases} \sin x + 2\sin(x+y+z) = 0 \\ \sin y + 3\sin(x+y+z) = 0 \\ \sin z + 4\sin(x+y+z) = 0 \end{cases}$$

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293 Given 1980 vectors in the plane, and there are some non-collinear among them. The sum of every 1979 vectors is collinear to the vector not included in that sum. Prove that the sum of all vectors equals to the zero vector.

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294 Let us denote with $S(n)$ the sum of all the digits of n.
a) Is there such an n that $n + S(n) = 1980$?
b) Prove that at least one of two arbitrary successive natural numbers is representable as $n + S(n)$ for some third number n.

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295 Some squares of the infinite sheet of cross-lined paper are red. Each 2×3 rectangle (of 6 squares) contains exactly two red squares. How many red squares can be in the 9×11 rectangle of 99 squares?

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296 An epidemic influenza broke out in the elves city. First day some of them were infected by the external source of infection and nobody later was infected by the external source. The elf is infected when visiting his ill friend. In spite of the situation every healthy elf visits all his ill friends every day. The elf is ill one day exactly, and has the immunity at least on the next day. There is no graftings in the city. Prove that
a) If there were some elves immunised by the external source on the first day, the epidemic influenza can continue arbitrary long time.
b) If nobody had the immunity on the first day, the epidemic influenza will stop some day.

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297 Let us denote with $P(n)$ the product of all the digits of n.
Consider the sequence $n_{k+1} = n_k + P(n_k)$.
Can it be unbounded for some n_1?

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298 Given equilateral triangle ABC. Some line, parallel to $[AC]$ crosses $[AB]$ and $[BC]$ in M and P points respectively. Let D be the centre of PMB triangle, E be the midpoint of the $[AP]$ segment. Find the angles of triangle DEC .

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299 Let the edges of rectangular parallelepiped be x, y and z ($x < y < z$).
Let $p = 4(x+y+z), s = 2(xy+yz+zx)$ and $d = \sqrt{x^2+y^2+z^2}$ be its perimeter, surface area and diagonal length, respectively.
Prove that $x < \frac{1}{3}\left(\frac{p}{4} - \sqrt{d^2 - \frac{s}{2}}\right)$ and $z > \frac{1}{3}\left(\frac{p}{4} - \sqrt{d^2 - \frac{s}{2}}\right)$

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300 The A set consists of integers only. Its minimal element is 1 and its maximal element is 100. Every element of A except 1 equals to the sum of two (may be equal) numbers being contained

in A.
What is the least possible number of elements in A?

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301 Prove that there is an infinite number of such numbers B that the equation $\lfloor x^3/2\rfloor + \lfloor y^3/2\rfloor = B$ has at least 1980 integer solutions (x, y).

($\lfloor z\rfloor$ denotes the greatest integer not exceeding z.)

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302 The edge $[AC]$ of the tetrahedron $ABCD$ is orthogonal to $[BC]$, and $[AD]$ is orthogonal to $[BD]$. Prove that the cosine of the angle between (AC) and (BD) lines is less than $|CD|/|AB|$.

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303 The number x from $[0, 1]$ is written as an infinite decimal fraction. Having rearranged its first five digits after the point we can obtain another fraction that corresponds to the number x_1. Having rearranged five digits of x_k from $(k + 1)$-th till $(k + 5)$-th after the point we obtain the number x_{k+1}.
a) Prove that the sequence x_i has limit.
b) Can this limit be irrational if we have started with the rational number?
c) Invent such a number, that always produces irrational numbers, no matter what digits were transposed.
