**All Soviet Union Mathematical Olympiad 1982**

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**327** Given two points $M$ and $K$ on the circumference with radius $r_1$ and centre $O_1$.
The circumference with radius $r_2$ and centre $O_2$ is inscribed in $MO_1K$ angle.
Find the $MO_1KO_2$ quadrangle area.

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**328** Every member, starting from the third one, of two sequences $\{a_n\}$ and $\{b_n\}$ equals to the sum of two preceding ones. First members are: $a_1 = 1, a_2 = 2, b_1 = 2, b_2 = 1$. How many natural numbers are encountered in both sequences (may be on the different places)?

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**329** a) Let $m$ and $n$ be natural numbers.
For some nonnegative integers $k_1, k_2, ..., k_n$ the number $2^{k_1} + 2^{k_2} + ... + 2^{k_n}$ is divisible by $(2^m - 1)$.
Prove that $n \geq m$.
b) Can You find a number, divisible by $111...1$ ($m$ times "$1$"), that has the sum of its digits less than $m$?

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**330** A nonnegative real number is written at every cube's vertex. The sum of those numbers equals to $1$. Two players choose in turn faces of the cube, but they cannot choose the face parallel to already chosen one (the first moves twice, the second – once). Prove that the first player can provide the number, at the common for three chosen faces vertex, to be not greater than $1/6$.

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**331** Once upon a time, three boys visited a library for the first time.
The first decided to visit the library every second day.
The second decided to visit the library every third day.
The third decided to visit the library every fourth day.
The librarian noticed, that the library doesn't work on Wednesdays.
The boys decided to visit library on Thursdays, if they have to do it on Wednesdays, but to restart the day counting in these cases.
They strictly obeyed these rules.
Some Monday later I met them all in that library.
What day of week was when they visited a library for the first time?

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**332** The parallelogram $ABCD$ isn't a diamond. The relation of the diagonal lengths $|AC|/|BD|$ equals to $k$. The $[AM)$ ray is symmetric to the $[AD)$ ray with respect to the $(AC)$ line. The $[BM)$ ray is symmetric to the $[BC)$ ray with respect to the $(BD)$ line. ($M$ point is those rays intersection.) Find the $|AM|/|BM|$ relation.

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**333** $3k$ points are marked on the circumference. They divide it onto $3k$ arcs. Some $k$ of them have length $1$, other $k$ of them have length $2$, the rest $k$ of them have length $3$. Prove that some two of the marked points are the ends of one diameter.

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**334** Given a point $M$ inside a right tetrahedron.
Prove that at least one tetrahedron edge is seen from the $M$ in an angle, that has a cosine not greater than $-1/3$.
(e.g. if $A$ and $B$ are the vertices, corresponding to that edge, $cos(\widehat{AMB}) \le -1/3$)

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**335** Three numbers $a, b, c$ belong to $[0, \pi/2]$ interval with $\cos a = a, \sin(\cos b) = b, \cos(\sin c) = c$.
Sort those numbers in increasing order.

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**336** The closed broken line $M$ has odd number of vertices – $A_1, A_2, ..., A_{2n+1}$ in sequence. Let us denote with $S(M)$ a new closed broken line with vertices $B_1, B_2, ..., B_{2n+1}$ – the midpoints of the first line links: $B_1$ is the midpoint of $[A_1A_2], ..., B_{2n+1}$ – of $[A_{2n+1}A_1]$. Prove that in a sequence $M_1 = S(M), ..., M_k = S(M_{k-1}), ...$ there is a broken line, homothetic to the $M$.

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**337** All the natural numbers from $1$ to $1982$ are gathered in an array in an arbitrary order in computer's memory.
The program looks through all the sequent pairs (first and second, second and third,...) and exchanges numbers in the pair, if the number on the lower place is greater than another.
Then the program repeats the process, but moves from another end of the array.
The number, that stand initially on the $100$-th place reserved its place.
Find that number.

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**338** Cucumber river in the Flower city has parallel banks with the distance between them $1$ metre.
It has some islands with the total perimeter $8$ metres.
Mr. Know-All claims that it is possible to cross the river in a boat from the arbitrary point,
and the trajectory will not exceed $3$ metres. Is he right?

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**339** There is a parabola $y = x^2$ drawn on the coordinate plane. The axes are deleted.
Can you restore them with the help of compass and ruler?

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**340** The square table $n \times n$ is filled by integers.
If the fields have common side, the difference of numbers in them doesn't exceed $1$.
Prove that some number is encountered not less than
a) not less than $[n/2]$ times ($[...]$ mean the whole part),
b) not less than $n$ times.

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**341** Prove that the following inequality is valid for the positive $x$:

$$2^{x^{1/12}} + 2^{x^{1/4}} \ge 2^{1+x^{1/6}}$$

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**342** What minimal number of numbers from the set $\{1, 2, ..., 1982\}$ should be deleted to provide the property:
*none of the remained numbers equals to the product of two other remained numbers*?

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**343** Every square on the infinite sheet of cross-lined paper contains some real number.
Prove that some square contains a number that does not exceed at least four of eight neighbouring numbers.

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**344** Given a sequence of real numbers $a_1, a_2, ..., a_n$.
Prove that it is possible to choose some of the numbers providing $3$ conditions:
a) not a triple of successive members is chosen,
b) at least one of every triple of successive members is chosen,
c) the absolute value of chosen numbers sum is not less that one sixth part of the initial numbers' absolute values sum.

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**345** Given the square table $n \times n$ with $(n-1)$ marked fields.
Prove that it is possible to move all the marked fields below the diagonal by moving rows and columns.

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**346** Prove that the following inequality holds for all real $a$ and natural $n$: $|a| \cdot |a-1| \cdot |a-2| \cdot ... \cdot |a-n| \geq \frac{n!F(a)}{2n}$ . $F(a)$ is the distance from $a$ to the closest integer.

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**347** Can You find three polynomials $P, Q, R$ of three variables $x, y, z$, providing the condition:
a)$P(x-y+z)^3 + Q(y-z-1)^3 + R(z-2x+1)^3 = 1$,
b)$P(x-y+z)^3 + Q(y-z-1)^3 + R(z-x+1)^3 = 1$,
for all $x, y, z$?

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**348** The $KLMN$ tetrahedron (triangle pyramid) vertices are situated inside or on the faces or on the edges of the $ABCD$ tetrahedron. Prove that $KLMN$ perimeter is less than $4/3$ $ABCD$ perimeter.

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