**All Soviet Union Mathematical Olympiad 1983**

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**349** Every cell of a $4 \times 4$ square grid net, has $1 \times 1$ size.
Is it possible to represent this net as a union of the following sets:
a) Eight broken lines of length five each?
b) Five broken lines of length eight each?

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**350** Three numbers were written with a chalk on the blackboard.
The following operation was repeated several times: One of the numbers was cleared and the sum of two other numbers, decreased by $1$, was written instead of it. The final set of numbers is $\{17, 1967, 1983\}$.
Is it possible to admit that the initial numbers were
a) $\{2, 2, 2\}$?
b) $\{3, 3, 3\}$?

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**351** Three disks touch pairwise from outside in the points $X, Y, Z$. Then the radiuses of the disks were expanded by $2/\sqrt{3}$ times, and the centres were reserved. Prove that the $XYZ$ triangle is completely covered by the expanded disks.

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**352** Find all the solutions of the system

$$\begin{cases} y^2 = x^3 - 3x^2 + 2x \\ x^2 = y^3 - 3y^2 + 2y \end{cases}$$

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**354** Natural number $k$ has $n$ digits in its decimal notation.
It was rounded up to tens, then the obtained number was rounded up to hundreds, and so on $(n-1)$ times.
Prove that the obtained number $m$ satisfies inequality $m < \frac{18k}{13}$.
(Examples of rounding: $191 \to 190 \to 200, 135 \to 140 \to 100.$)

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**355** The point $D$ is the midpoint of the side $[AB]$ of the triangle $ABC$ . The points $E$ and F belong to $[AC]$ and $[BC]$ respectively. Prove that the $DEF$ triangle area does not exceed the sum of the $ADE$ and $BDF$ triangles areas.

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**356** The sequences $a_n$ and $b_n$ members are the last digits of $[\sqrt{10}^n]$ and $[\sqrt{2}^n]$ respectively (here $[...]$ denotes the whole part of a number). Are those sequences periodical?

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**357** Two acute angles $a$ and $b$ satisfy condition $\sin^2 a + \sin^2 b = \sin(a + b)$
Prove that $a + b = \pi/2$.

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**358** The points $A_1, B_1, C_1, D_1$ and $A_2, B_2, C_2, D_2$ are orthogonal projections of the $ABCD$ tetrahedron vertices on two planes. Prove that it is possible to move one of the planes to provide the parallelness of $(A_1A_2), (B_1B_2), (C_1C_2)$ and $(D_1D_2)$ lines.

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**359** The pupil is training in the square equation solution.
Having the recurrent equation solved, he stops, if it doesn't have two roots, or solves the next equation, with the free coefficient equal to the greatest root, the coefficient at $x$ equal to the least root, and the coefficient at $x^2$ equal to $1$. Prove that the process cannot be infinite.
What maximal number of the equations he will have to solve?

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**360** Given natural $n, m, k$. It is known that $m^n$ is divisible by $n^m$, and $n^k$ is divisible by $k^n$.
Prove that $m^k$ is divisible by $k^m$.

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**361** The Abba tribe language alphabet contains two letters only. Not a word of this language is a beginning of another word. Can this tribe vocabulary contain $3$ four-letter, $10$ five-letter, $30$ six-letter and $5$ seven-letter words?

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**362** Can You fill the squares of the infinite cross-lined paper with integers so, that the sum of the numbers in every $4 \times 6$ fields rectangle would be
a) $10$?
b) $1$?

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**363** The points $A_1, B_1, C_1$ belong to $[BC], [CA], [AB]$ sides of the $ABC$ triangle respectively. The $[AA_1], [BB_1], [CC_1]$ segments split the $ABC$ onto $4$ smaller triangles and $3$ quadrangles. It is known, that the smaller triangles have the same area. Prove that the quadrangles have equal areas. What is the quadrangle area, it the small triangle has the unit area?

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**364** The kindergarten group is standing in the column of pairs. The number of boys equals the number of girls in each of the two columns. The number of mixed (boy and girl) pairs equals to the number of the rest pairs. Prove that the total number of children in the group is divisible by eight.

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**365** One side of the rectangle is $1$ cm. It is known that the rectangle can be divided by two orthogonal lines onto four rectangles, and each of the smaller rectangles has the area not less than $1$ square centimetre, and one of them is not less than $2$ square centimetres. What is the least possible length of another side of big rectangle?

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**366** Given a point $O$ inside $ABC$ triangle.
Prove that $S_A * \overrightarrow{OA} + S_B * \overrightarrow{OB} + S_C * \overrightarrow{OC} = \overrightarrow{0}$,
where $S_A, S_B, S_C$ denote $BOC, COA, AOB$ triangles areas respectively.

**367** Given $(2m+1)$ different integers, each absolute value is not greater than $(2m-1)$.
Prove that it is possible to choose three numbers among them, with their sum equal to zero.

**368** The points $D, E, F$ belong to the sides $(AB), (BC)$ and $(CA)$ of the $ABC$ triangle respectively (but they are not vertices). Let us denote with $d_0, d_1, d_2$, and $d_3$ the maximal side length of the $DEF, DEA, DBF, CEF$, triangles respectively. Prove that $d_0 \geq \frac{\sqrt{3}}{2} min\{d_1, d_2, d_3\}$. When the equality takes place?

**369** The $M$ set consists of $k$ non-intersecting segments on the line. It is possible to put an arbitrary segment shorter than $1$ cm on the line in such a way, that his ends will belong to $M$. Prove that the total sum of the segment lengths is not less than $1/k$ cm.

**370** The infinite decimal notation of the real number $x$ contains all the digits. Let $u_n$ be the number of different $n$-digit segments encountered in $x$ notation. Prove that if for some $n$, $u_n \leq (n+8)$, than $x$ is a rational number.

 