All Soviet Union Mathematical Olympiad 1987

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441 Ten sportsmen have taken part in a table-tennis tournament (each pair has met once only, no draws).
Let xi be the number of i-th player victories, yi – losses.
Prove that $x_1^2 + ... + x_{10}^2 = y_1^2 + ... + y_{10}^2$

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442 It is known that, having 6 weighs, it is possible to balance the scales with loads, which weights are successing natural numbers from 1 to 63. Find all such sets of weighs.

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443 Given a regular heptagon $A_1...A_7$. Prove that $\frac{1}{|A_1A_5|} + \frac{1}{|A_1A_3|} = \frac{1}{|A_1A_7|}$.

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444 The "Sea battle" game.
a) You are trying to find the 4-field ship – a rectangle $1x4$, situated on the $7x7$ playing board. You are allowed to ask a question, whether it occupies the particular field or not. How many questions is it necessary to ask to find that ship surely?
b) The same question, but the ship is a connected (i.e. its fields have common sides) set of 4 fields.

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444 Prove that $1^{1987} + 2^{1987} + ... + n^{1987}$ is divisible by $n + 2$.

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446 An L is an arrangement of 3 adjacent unit squares formed by deleting one unit square from a 2×2 square.
a) How many Ls can be placed on an 8×8 board (with no interior points overlapping)?
b) Show that if any one square is deleted from a 1987×1987 board, then the remaining squares can be covered with Ls (with no interior points overlapping).

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447 Three lines are drawn parallel to the sides of the triangles in the opposite to the vertex, not belonging to the side, part of the plane. The distance from each side to the corresponding line equals the length of the side. Prove that six intersection points of those lines with the continuations of the sides are situated on one circumference.

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448 Given two closed broken lines in the plane with odd numbers of edges. All the lines, containing those edges are different, and not a triple of them intersects in one point. Prove that it is possible to chose one edge from each line such, that the chosen edges will be the opposite sides of a convex quadrangle.

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449 Find a set of five different relatively prime natural numbers such, that the sum of an arbitrary subset is a composite number.

450 Given a convex pentagon. The angles ABC and ADE are equal. The angles AEC and ADB are equal too. Prove that the angles BAC and DAE are equal also.

451 Prove such a, that all the numbers $\cos a, \cos 2a, \cos 4a, ..., \cos(2^n a)$ are negative.

452 The positive numbers a, b, c, A, B, C satisfy a condition $a + A = b + B = c + C = k$.
Prove that $aB + bC + cA \leq k^2$.

453 Each field of the 1987×1987 board is filled with numbers, which absolute value is not greater than one. The sum of all the numbers in every 2×2 square equals 0. Prove that the sum of all the numbers is not greater than 1987.

454 The B vertex of the ABC angle lies out the circle, and the $[BA)$ and $[BC)$ beams intersect it. The K point belongs to the intersection of the $[BA)$ beam and the circumference. The KP chord is orthogonal to the angle ABC bisector. The (KP) line intersects the BC beam in the M point. Prove that the $[PM]$ segment is twice as long as the distance from the circle centre to the angle ABC bisector.

455 Two players are writting in turn natural numbers not exceeding p.
The rules forbid to write the divisors of the numbers already having been written.
Those who cannot make his move looses.
a) Who, and how, can win if $p = 10$?
b) Who wins if $p = 1000$?

456 Every evening uncle Chernomor (see Pushkin's tales) appoints either 9 or 10 of his 33 ”knights” in the ”night guard”. When it can happen, for the first time, that every knight has been on duty the same number of times?

457 Some points with the integer coordinates are marked on the coordinate plane. Given a set of nonzero vectors. It is known, that if you apply the beginnings of those vectors to the arbitrary marked point, than there will be more marked ends of the vectors, than not marked. Prove that there is infinite number of marked points.

458 The convex n-gon ($n \geq 5$) is cut along all its diagonals.
Prove that there are at least a pair of parts with the different areas.

459 The T_0 set consists of all the numbers, representable as $(2k)!, k = 0, 1, 2, ..., n, ...$.
The T_m set is obtained from T_{m-1} by adding all the finite sums of different numbers, that belong to T_{m-1}.
Prove that there is a natural number, that doesn't belong to T_{1987}.

460 The plot of the $y = f(x)$ function, being rotated by the (right) angle around the $(0,0)$ point is not changed.
a) Prove that the equation $f(x) = x$ has the unique solution.
b) Give an example of such a function.

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461 All the faces of a convex polyhedron are the triangles.
Prove that it is possible to paint all its edges in red and blue colour in such a way, that it is possible to move from the arbitrary vertex to every vertex along the blue edges only and along the red edges only.

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462 Prove that for every natural n the following inequality is held: $(2n+1)^n \geq (2n)^n + (2n-1)^n$.
