Moscow Mathematical Olympiad 1939

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–	tour 1
043	Solve the system $\begin{cases} 3xyz-x^3-y^3-z^3=b^3 \\ x+y+z=2b \\ x^2+y^2-z^2=b^2 \end{cases}$
044	Prove that $cos \frac{2\pi}{5} + cos \frac{4\pi}{5} = -\frac{1}{2}$.
045	Consider points A, B, C. Draw a line through A so that the sum of distances from B and C to this line is equal to the length of a given segment.
046	Solve the equation $\sqrt{a-\sqrt{a+x}} = x$ for x.
047	Prove that for any triangle the bisector lies between the median and the height drawn from the same vertex.
–	tour 2
048	Factor $a^{10}+a^5+1$ into nonconstant polynomials with integer coefficients
049	Let the product of two polynomials of a variable x with integer coefficients be a polynomial with even coefficients not all of which are divisible by 4. Prove that all the coefficients of one of the polynomials are even and that at least one of the coefficients of the other polynomial is odd.
050	Given two points A and B and a circle, find a point X on the circle so that points C and D at which lines AX and BX intersect the circle are the endpoints of the chord CD parallel to a given line MN.
051	Find the remainder after division of $10^{10}+10^{10^2}+10^{10^3}+...+10^{10^{10}}$ by 7.
052	Consider a regular pyramid and a perpendicular to its base at an arbitrary point P. Prove that the sum of the lengths of the segments connecting P to the intersection points of the perpendicular with the planes of the pyramids faces does not depend on the location of P.
053	What is the greatest number of parts that 5 spheres can divide the space into?