

Moscow Mathematical Olympiad 1940

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by parmenides51

– tour 1

054 Factor $(b - c)^3 + (c - a)^3 + (a - b)^3$.

055 It takes a steamer 5 days to go from Gorky to Astrakhan downstream the Volga river and 7 days upstream from Astrakhan to Gorky. How long will it take for a raft to float downstream from Gorky to Astrakhan?

056 How many zeros does $100!$ have at its end in the usual decimal representation?

057 Draw a circle that has a given radius R and is tangent to a given line and a given circle. How many solutions does this problem have?

058 Solve the system
$$\begin{cases} (x^3 + y^3)(x^2 + y^2) = 2b^5 \\ x + y = b \end{cases}$$

059 Consider all positive integers written in a row: 123456789101112131415...
Find the 206788-th digit from the left.

060 Construct a circle equidistant from four points on a plane. How many solutions are there?

061 Given two lines on a plane, find the locus of all points with the difference between the distance to one line and the distance to the other equal to the length of a given segment.

062 Find all 3-digit numbers \overline{abc} such that $\overline{abc} = a! + b! + c!$.

– tour 2

062- Find a four-digit number that is perfect square and such that the first two digits are the same and the last two as well.

063 Points A, B, C are vertices of an equilateral triangle inscribed in a circle. Point D lies on the shorter arc \widehat{AB} . Prove that $AD + BD = DC$.

064 How does one tile a plane, without gaps or overlappings, with the tiles equal to a given irregular quadrilateral?

065 How many pairs of integers x, y are there between 1 and 1000 such that $x^2 + y^2$ is divisible by 7?

066 * Given an infinite cone. The measure of its unfolding angle is equal to α . A curve on the cone is represented on any unfolding by the union of line segments. Find the number of the curves self-intersections.

067 Which is greater: $300!$ or 100^{300} ?

068 The center of the circle circumscribing $\triangle ABC$ is mirrored through each side of the triangle and three points are obtained: O_1, O_2, O_3 . Reconstruct $\triangle ABC$ from O_1, O_2, O_3 if everything else is erased.

069 Let a_1, \dots, a_n be positive numbers. Prove the inequality:

$$\frac{a_1}{a_2} + \frac{a_2}{a_3} + \frac{a_3}{a_4} + \dots + \frac{a_{n-1}}{a_n} + \frac{a_n}{a_1} \geq n$$

070 How many positive integers x less than 10000 are there such that $2^x - x^2$ is divisible by 7?
