Moscow Mathematical Olympiad 1941

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by parmenides51

–	tour 1
071	Construct a triangle given its height and median both from the same vertex and the radius of the circumscribed circle.
072	Find the number $\overline{523abc}$ divisible by $7, 8$ and 9.
073	Given a quadrilateral, the midpoints A, B, C, D of its consecutive sides, and the midpoints of its diagonals, P and Q. Prove that $\triangle BCP = \triangle ADQ$.
074	A point P lies outside a circle. Consider all possible lines drawn through P so that they intersect the circle. Find the locus of the midpoints of the chords segments the circle intercepts on these lines.
075	Prove that 1 plus the product of any four consecutive integers is a perfect square.
076	On the sides of a parallelogram, squares are constructed outwards. Prove that the centers of these squares are vertices of a square.
077	A polynomial $P(x)$ with integer coefficients takes odd values at $x = 0$ and $x = 1$. Prove that $P(x)$ has no integer roots.
078	Given points M and N, the bases of heights AM and BN of $\triangle ABC$ and the line to which the side AB belongs. Construct $\triangle ABC$.
079	Solve the equation: $\|x + 1\| - \|x\| + 3\|x - 1\| - 2\|x - 2\| = x + 2$.
080	How many roots does equation $\sin x = \frac{x}{100}$ have?
–	tour 2
081	a) Prove that it is impossible to divide a rectangle into five squares of distinct sizes. b) Prove that it is impossible to divide a rectangle into six squares of distinct sizes.
082	* Given $\triangle ABC$, divide it into the minimal number of parts so that after being flipped over these parts can constitute the same $\triangle ABC$.

083 Consider $\triangle ABC$ and a point M inside it. We move M parallel to BC until M meets CA, then parallel to AB until it meets BC, then parallel to CA, and so on. Prove that M traverses a self-intersecting closed broken line and find the number of its straight segments.

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084 a) Find an integer a for which $(x-a)(x-10)+1$ factors in the product $(x+b)(x+c)$ with integers b and c.
b) Find nonzero and nonequal integers a, b, c so that $x(x-a)(x-b)(x-c)+1$ factors into the product of two polynomials with integer coefficients.

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085 Prove that the remainder after division of the square of any prime $p > 3$ by 12 is equal to 1.

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086 Given three points H_1, H_2, H_3 on a plane. The points are the reflections of the intersection point of the heights of the triangle $\triangle ABC$ through its sides. Construct $\triangle ABC$.

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087 On a plane, several points are chosen so that a disc of radius 1 can cover every 3 of them. Prove that a disc of radius 1 can cover all the points.

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088 Solve in integers the equation $x+y = x^2 - xy + y^2$.

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089 Given two skew perpendicular lines in space, find the set of the midpoints of all segments of given length with the endpoints on these lines.

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090 Construct a right triangle, given two medians drawn to its legs.
