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by dcouchman, ThE-dArK-lOrD

– Day 1

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1 Determine all polynomial $P(x) \in \mathbb{R}[x]$ satisfying the following two conditions:
(a) $P(2017) = 2016$ and
(b) $(P(x) + 1)^2 = P(x^2 + 1)$ for all real number x.

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2 Let $ABCDE$ be a regular pentagon with center M. A point $P \neq M$ is chosen on the line segment MD. The circumcircle of ABP intersects the line segment AE in A and Q and the line through P perpendicular to CD in P and R.
Prove that AR and QR are of the same length.

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3 Anna and Berta play a game in which they take turns in removing marbles from a table. Anna takes the first turn. When at the beginning of the turn there are $n \geq 1$ marbles on the table, then the player whose turn it is removes k marbles, where $k \geq 1$ either is an even number with $k \leq \frac{n}{2}$ or an odd number with $\frac{n}{2} \leq k \leq n$. A player win the game if she removes the last marble from the table.
Determine the smallest number $N \geq 100000$ such that Berta can enforce a victory if there are exactly N marbles on the tale in the beginning.

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4 Find all pairs (a, b) of non-negative integers such that $2017^a = b^6 - 32b + 1$.
