**Moscow Mathematical Olympiad 1946**

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by parmenides51

**–** tour 1

**106** What is the largest number of acute angles that a convex polygon can have?

**107** Given points $A, B, C$ on a line, equilateral triangles $ABC_1$ and $BCA_1$ constructed on segments $AB$ and $BC$, and midpoints $M$ and $N$ of $AA_1$ and $CC_1$, respectively. Prove that $\triangle\ BMN$ is equilateral. (We assume that $B$ lies between $A$ and $C$, and points $A_1$ and $C_1$ lie on the same side of line $AB$)

**108** Find a four-digit number such that the remainders after its division by $131$ and $132$ are $112$ and $98$, respectively.

**109** Solve the system of equations: $$\begin{cases} x_1+x_2+x_3=6 \\ x_2+x_3+x_4=9 \\ x_3+x_4+x_5=3 \\ x_4+x_5+x_6=-3 \\ x_5+x_6+x_7=-9 \\ x_6+x_7+x_8=-6 \\ x_7+x_8+x_1=-2 \\ x_8+x_1+x_2=2 \end{cases}$$

**110** Prove that after completing the multiplication and collecting the terms $(1-x+x^2-x^3+...-x^{99}+x^{100})(1+x+x^2+...+x^{99}+x^{100})$
has no monomials of odd degree.

**111** Given two intersecting planes $\alpha$ and $\beta$ and a point $A$ on the line of their intersection. Prove that of all lines belonging to $\alpha$ and passing through $A$ the line which is perpendicular to the intersection line of $\alpha$ and $\beta$ forms the greatest angle with $\beta$.

**112** Through a point $M$ inside an angle $a$ line is drawn. It cuts off this angle a triangle of the least possible area. Prove that $M$ is the midpoint of the segment on this line that the angle intercepts.

**113** Prove that $n^2+3n+5$ is not divisible by $121$ for any positive integer $n$.

**114** Prove that for any positive integer $n$ the following identity holds $\frac{(2n)!}{n!} = 2^n \cdot (2n-1)!!$

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**115** Prove that if $\alpha$ and $\beta$ are acute angles and $\alpha$ ¡ $\beta$ , then $\frac{tan \alpha}{\alpha} < \frac{tan \beta}{\beta}$

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**–** tour 2

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**116** a) Two seventh graders and several eightth graders take part in a chess tournament. The two seventh graders together scored eight points. The scores of eightth graders are equal. How many eightth graders took part in the tournament?
b) Ninth and tenth graders participated in a chess tournament. There were ten times as many tenth graders as ninth graders. The total score of tenth graders was $4.5$ times that of the ninth graders. What was the ninth graders score?

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**117** Prove that for any integers $x$ and $y$ we have $x^5 + 3x^4y - 5x^3y^2 - 15x^2y^3 + 4xy^4 + 12y^5 \ne 33$.

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**119** On the legs of $\angle AOB$, the segments $OA$ and $OB$ lie, $OA > OB$. Points $M$ and $N$ on lines $OA$ and $OB$, respectively, are such that $AM = BN = x$. Find $x$ for which the length of $MN$ is minimal.

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**119** Towns $A_1, A_2, ..., A_{30}$ lie on line $MN$. The distances between the consecutive towns are equal. Each of the towns is the point of origin of a straight highway. The highways are on the same side of $MN$ and form the following angles with it:
`https://cdn.artofproblemsolving.com/attachments/a/f/6cfcac497bdd729b966705f1060bd4b1caba2`
`png`

Thirty cars start simultaneously from these towns along the highway at the same constant speed. Each intersection has a gate. As soon as the first (in time, not in number) car passes the intersection the gate closes and blocks the way for all other cars approaching this intersection. Which cars will pass all intersections and which will be stopped?

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**120** a) A bus network is organized so that:
1) one can reach any stop from any other stop without changing buses;
2) every pair of routes has a single stop at which one can change buses;
3) each route has exactly three stops?
How many bus routes are there?

b) A town has $57$ bus routes. How many stops does each route have if it is known that
1) one can reach any stop from any other stop without changing buses;
2) for every pair of routes there is a single stop where one can change buses;
3) each route has three or more stops?

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**121** Given the Fibonacci sequence $0, 1, 1, 2, 3, 5, 8, ...,$ ascertain whether among its first $(10^8 + 1)$ terms there is a number that ends with four zeros.

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**122** On the sides $PQ, QR, RP$ of $\triangle PQR$ segments $AB, CD, EF$ are drawn.
Given a point $S_0$ inside triangle $\triangle PQR$, find the locus of points $S$ for which the sum of the areas of triangles $\triangle SAB, \triangle SCD$ and $\triangle SEF$ is equal to the sum of the areas of triangles $\triangle S_0AB, \triangle S_0CD, \triangle S0EF$.
Consider separately the case $\frac{AB}{PQ} = \frac{CD}{QR} = \frac{EF}{RP}$ .