Moscow Mathematical Olympiad 1947

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by parmenides51

–	tour 1
123	Find the remainder after division of the polynomial $x + x^3 + x^9 + x^{27} + x^{81} + x^{243}$ by $x - 1$.
124	a) Prove that of 5 consecutive positive integers one that is relatively prime with the other 4 can always be selected. b) Prove that of 10 consecutive positive integers one that is relatively prime with the other 9 can always be selected.
125	Find the coefficients of x^{17} and x^{18} after expansion and collecting the terms of $(1+x^5+x^7)^{20}$.
126	Given a convex pentagon $ABCDE$, prove that if an arbitrary point M inside the pentagon is connected by lines with all the pentagons vertices, then either one or three or five of these lines cross the sides of the pentagon opposite the vertices they pass.
127	Point O is the intersection point of the heights of an acute triangle $\triangle ABC$. Prove that the three circles which pass: a) through O, A, B, b) through O, B, C, and c) through O, C, A, are equal
128	Find the coefficient of x^2 after expansion and collecting the terms of the following expression (there are k pairs of parentheses): $((...(((x - 2)^2 - 2)^2 - 2)^2 - ... - 2)^2 - 2)^2$.
129	How many squares different in size or location can be drawn on an 8×8 chess board? Each square drawn must consist of whole chess boards squares.
130	Which of the polynomials, $(1 + x^2 - x^3)^{1000}$ or $(1 - x^2 + x^3)^{1000}$, has the greater coefficient of x^{20} after expansion and collecting the terms?
131	Calculate (without calculators, tables, etc.) with accuracy to 0.00001 the product $\left(1 - \frac{1}{10}\right)\left(1 - \frac{1}{10^2}\right) ... \left(1 -\right.$
132	Given line AB and point M. Find all lines in space passing through M at distance d.
–	tour 2

133 Twenty cubes of the same size and appearance are made of either aluminum or of heavier duralumin. How can one find the number of duralumin cubes using not more than 11 weighings on a balance without weights? (We assume that all cubes can be made of aluminum, but not all of duralumin.)

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134 How many digits are there in the decimal expression of 2^{100} ?

135- Position the 4 points on plane so that when measuring of all pairwise distances between them, it turned out only two different numbers. Find all such locations.

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135 a) Given 5 points on a plane, no three of which lie on one line.
Prove that four of these points can be taken as vertices of a convex quadrilateral.
b) Inside a square, consider a convex quadrilateral and inside the quadrilateral, take a point A.
It so happens that no three of the 9 points the vertices of the square, of the quadrilateral and A lie on one line.
Prove that 5 of these points are vertices of a convex pentagon.

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136 Prove that no convex 13-gon can be cut into parallelograms.

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137 a) 101 numbers are selected from the set $1, 2, ..., 200$.
Prove that among the numbers selected there is a pair in which one number is divisible by the other.
b) One number less than 16, and 99 other numbers are selected from the set $1, 2, ..., 200$.
Prove that among the selected numbers there are two such that one divides the other.

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138 In space, n wire triangles are situated so that any two of them have a common vertex and each vertex is the vertex of k triangles. Find all n and k for which this is possible.

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139 In the numerical triangle ....1.............. ....$1...1...1$......... ......$1...2...3...2...1$.... .$1...3...6...7...6...3...1$ ....
each number is equal to the sum of the three nearest to it numbers from the row above it; if the number is at the beginning or at the end of a row then it is equal to the sum of its two nearest numbers or just to the nearest number above it (the lacking numbers above the given one are assumed to be zeros). Prove that each row, starting with the third one, contains an even number.

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140 Prove that if the four faces of a tetrahedron are of the same area they are equal.
