Moscow Mathematical Olympiad 1948

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by parmenides51

–	tour 1
141	The sum of the reciprocals of three positive integers is equal to 1. What are all the possible such triples?
142	Find all possible arrangements of 4 points on a plane, so that the distance between each pair of points is equal to either a or b. For what ratios of $a : b$ are such arrangements possible?
143	On a plane, n straight lines are drawn. Two domains are called *adjacent* if they border by a line segment. Prove that the domains into which the plane is divided by these lines can be painted two colors so that no two *adjacent* domains are of the same color.
144	Prove that if $\frac{2^n-2}{n}$ is an integer, then so is $\frac{2^{2^n-1}-2}{2^n-1}$.
145	Without tables and such, prove that $\frac{1}{\log_2 \pi}+\frac{1}{\log_5 \pi}>2$
146	Consider two triangular pyramids $ABCD$ and $A'BCD$, with a common base BCD, and such that A' is inside $ABCD$. Prove that the sum of planar angles at vertex A' of pyramid $A'BCD$ is greater than the sum of planar angles at vertex A of pyramid $ABCD$.

147	Consider a circle and a point A outside it. We start moving from A along a closed broken line consisting of segments of tangents to the circle (the segment itself should not necessarily be tangent to the circle) and terminate back at A. (On the links of the broken line are solid.) We label parts of the segments with a plus sign if we approach the circle and with a minus sign otherwise. Prove that the sum of the lengths of the segments of our path, with the signs given, is zero.
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–	tour 2
148	a) Find all positive integer solutions of the equation $x^y = y^x$ $(x \ne y)$. b) Find all positive rational solutions of the equation $x^y = y^x$ $(x \ne y)$.
149	Let R and r be the radii of the circles circumscribed and inscribed, respectively, in a triangle. Prove that $R \ge 2r$, and that $R = 2r$ only for an equilateral triangle.

150 Can a figure have a greater than 1 and finite number of centers of symmetry?

151 The distance between the midpoints of the opposite sides of a convex quadrilateral is equal to a half sum of lengths of the other two sides. Prove that the first pair of sides is parallel.

152 a) Two legs of an angle α on a plane are mirrors. Prove that after several reflections in the mirrors any ray leaves in the direction opposite the one from which it came if and only if $\alpha = \frac{90^o}{n}$ for an integer n. Find the number of reflections.
b) Given three planar mirrors in space forming an octant (trihedral angle with right planar angles), prove that any ray of light coming into this mirrored octant leaves it, after several reflections in the mirrors, in the direction opposite to the one from which it came. Find the number of reflections.

153 * What is the radius of the largest possible circle inscribed into a cube with side a?

154 How many different integer solutions to the inequality $|x| + |y| < 100$ are there?

155 What is the greatest number of rays in space beginning at one point and forming pairwise obtuse angles?