**Moscow Mathematical Olympiad 1949**

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by parmenides51

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| – | tour 1 |
| **156** | Prove that $27195^8 - 10887^8 + 10152^8$ is divisible by $26460$. |
| **157** | a) Prove that if a planar polygon has several axes of symmetry, then all of them intersect at one point.<br>b) A finite solid body is symmetric about two distinct axes. Describe the position of the symmetry planes of the body. |
| **158** | a) Prove that $x^2 + y^2 + z^2 = 2xyz$ for integer $x, y, z$ only if $x = y = z = 0$.<br>b) Find integers $x, y, z, u$ such that $x^2 + y^2 + z^2 + u^2 = 2xyzu$. |
| **159** | Consider a closed broken line of perimeter $1$ on a plane. Prove that a disc of radius $\frac{1}{4}$ can cover this line. |
| **160** | Prove that for any triangle the circumscribed circle divides the line segment connecting the center of its inscribed circle with the center of one of the exscribed circles in halves. |
| **161** | Find the real roots of the equation $x^2 + 2ax + \frac{1}{16} = -a + \sqrt{a^2 + x - \frac{1}{16}}$, $\left(0 < a < \frac{1}{4}\right)$ . |
| **162** | Given a set of $4n$ positive numbers such that any distinct choice of ordered foursomes of these numbers constitutes a geometric progression. Prove that at least $4$ numbers of the set are identical. |
| **163** | Prove that if opposite sides of a hexagon are parallel and the diagonals connecting opposite vertices have equal lengths, a circle can be circumscribed around the hexagon. |
| – | tour 2 |
| **164** | There are $12$ points on a circle. Four checkers, one red, one yellow, one green and one blue sit at neighboring points. In one move any checker can be moved four points to the left or right, onto the fifth point, if it is empty. If after several moves the checkers appear again at the four original points, how might their order have changed? |
| **165** | Consider two triangles, $ABC$ and $DEF$, and any point $O$. We take any point $X$ in $\triangle\ ABC$ and any point $Y$ in $\triangle\ DEF$ and draw a parallelogram $OXYZ$. Prove that the locus of all possible |

points $Z$ form a polygon. How many sides can it have? Prove that its perimeter is equal to the sum of perimeters of the original triangles.

**166** Consider $13$ weights of integer mass (in grams). It is known that any $6$ of them may be placed onto two pans of a balance achieving equilibrium. Prove that all the weights are of equal mass.

**167** The midpoints of alternative sides of a hexagon are connected by line segments. Prove that the intersection points of the medians of the two triangles obtained coincide.

**168** Prove that some (or one) of any $100$ integers can always be chosen so that the sum of the chosen integers is divisible by $100$.

**169** Construct a convex polyhedron of equal bricks shown in Figure.
`https://cdn.artofproblemsolving.com/attachments/6/6/75681a90478f978665b6874d0c0c9441ea3bc`
`gif`

**170** What is a centrally symmetric polygon of greatest area one can inscribe in a given triangle?

**171** * Prove that a number of the form $2^n$ for a positive integer $n$ may begin with any given combination of digits.

**172** Two squares are said to be *juxtaposed* if their intersection is a point or a segment. Prove that it is impossible to *juxtapose* to a square more than eight non-overlapping squares of the same size.