Serbia Team Selection Test 2019

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– Day 1

P1 a) Given 2019 different integers wich have no odd prime divisor less than 37, prove there exists two of these numbers such that their sum has no odd prime divisor less than 37.

b)Does the result hold if we change 37 to 38 ?

P2 Given triangle $\triangle ABC$ with $AC \neq BC$,and let D be a point inside triangle such that $\measuredangle ADB = 90^\circ + \frac{1}{2}\measuredangle ACB$.Tangents from C to the circumcircles of $\triangle ABC$ and $\triangle ADC$ intersect AB and AD at P and Q , respectively.Prove that PQ bisects the angle $\measuredangle BPC$.

P3 It is given n a natural number and a circle with circumference n. On the circle, in clockwise direction, numbers $0, 1, 2, \dots n-1$ are written, in this order and in the same distance to each other. Every number is colored red or blue, and there exists a non-zero number of numbers of each color. It is known that there exists a set $S \subsetneq \{0, 1, 2, \dots n-1\}, |S| \geq 2$, for wich it holds: if $(x, y), x < y$ is a circle sector whose endpoints are of distinct colors, whose distance $y - x$ is in S, then y is in S.

Prove that there is a divisor d of n different from 1 and n for wich holds: if $(x, y), x < y$ are different points of distinct colors, such that their distance is divisible by d, then both x, y are divisible by d.

– Day 2

P4 A trader owns horses of 3 races, and exacly b_j of each race (for $j = 1, 2, 3$). He want to leave these horses heritage to his 3 sons. He knowns that the boy i for horse j (for $i, j = 1, 2, 3$) would pay a_{ij} golds, such that for distinct i, j holds holds $a_{ii} > a_{ij}$ and $a_{jj} > a_{ij}$.

Prove that there exists a natural number n such that whenever it holds $\min\{b_1, b_2, b_3\} > n$, trader can give the horses to their sons such that after getting the horses each son values his horses more than the other brother is getting, individually.

P5 Solve the equation in nonnegative integers:

$$2^x = 5^y + 3$$

P6 A *figuric* is a convex polyhedron with $26^{5^{2019}}$ faces. On every face of a figuric we write down a number. When we throw two figurics (who don't necessarily have the same set of numbers on their sides) into the air, the figuric which falls on a side with the greater number wins; if this number is equal for both figurics, we repeat this process until we obtain a winner. Assume that a figuric has an equal probability of falling on any face. We say that one figuric rules over another if when throwing these figurics into the air, it has a strictly greater probability to win than the other figuric (it can be possible that given two figurics, no figuric rules over the other). Milisav and Milojka both have a blank figuric. Milisav writes some (not necessarily distinct) positive integers on the faces of his figuric so that they sum up to $27^{5^{2019}}$. After this, Milojka also writes positive integers on the faces of her figuric so that they sum up to $27^{5^{2019}}$. Is it always possible for Milojka to create a figuric that rules over Milisav's?

Proposed by Bojan Basic