1995 Korean Math Olympiad

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– Day 1

Problem 1 For any positive integer m,show that there exist integers a, b satisfying $|a| \leq m$, $|b| \leq m$, $0 < a + b\sqrt{2} \leq \frac{1+\sqrt{2}}{m+2}$

Problem 2 find all functions from the nonegative integers into themselves, such that: $2f(m^2+n^2) = f^2(m) + f^2(n)$ and for $m \geq n$ $f(m^2) \geq f(n^2)$.

Problem 3 Let ABC be an equilateral triangle of side 1, D be a point on BC, and r_1, r_2 be the inradii of triangles ABD and ADC. Express $r_1 r_2$ in terms of $p = BD$ and find the maximum of $r_1 r_2$.

– Day 2

Day 1 Let O and R be the circumcenter and circumradius of a triangle ABC, and let P be any point in the plane of the triangle. The perpendiculars PA_1, PB_1, PC_1 are drawn from P on BC, CA, AB. Express $S_{A_1B_1C_1}/S_{ABC}$ in terms of R and $d = OP$, where S_{XYZ} is the area of $\triangle XYZ$.

Day 2 Let a, b be integers and p be a prime number such that:
(i) p is the greatest common divisor of a and b;
(ii) p^2 divides a.
Prove that the polynomial $x^{n+2} + ax^{n+1} + bx^n + a + b$ cannot be decomposed into the product of two polynomials with integer coefficients and degree greater than 1.

Day 3 Let m, n be positive integers with $1 \leq n < m$. A box is locked with several padlocks which must all be opened to open the box, and which all have different keys. The keys are distributed among m people. Suppose that among these people, no n can open the box, but any $n+1$ can open it. Find the smallest possible number l of locks and then the total number of keys for which this is possible.