www.artofproblemsolving.com/community/c909771

by BigSams, k.vasilev

– Day 1

1 Let $f(x)=x^2+bx+1$, where b is a real number. Find the number of integer solutions to the inequality $f(f(x)+x)<0$.

2 Let ABC be an acute triangle with orthocenter H and circumcenter O. Let the intersection points of the perpendicular bisector of CH with AC and BC be X and Y respectively. Lines XO and YO cut AB at P and Q respectively. If $XP+YQ=AB+XY$, determine $\measuredangle OHC$.

3 Find all real numbers a, which satisfy the following condition:

For every sequence $a_1,a_2,a_3,\ldots$ of pairwise different positive integers, for which the inequality $a_n\leq an$ holds for every positive integer n, there exist infinitely many numbers in the sequence with sum of their digits in base 4038, which is not divisible by 2019.

– Day 2

4 Determine all positive integers d, such that there exists an integer $k\geq 3$, such that
One can arrange the numbers $d,2d,\ldots,kd$ in a row, such that the sum of every two consecutive of them is a perfect square.

5 Let P be a $2019-$gon, such that no three of its diagonals concur at an internal point. We will call each internal intersection point of diagonals of P a knot. What is the greatest number of knots one can choose, such that there doesn't exist a cycle of chosen knots? (Every two adjacent knots in a cycle must be on the same diagonal and on every diagonal there are at most two knots from a cycle.)

6 Let $ABCDEF$ be an inscribed hexagon with

$$AB.CD.EF=BC.DE.FA$$

Let B_1 be the reflection point of B with respect to AC and D_1 be the reflection point of D with respect to CE, and finally let F_1 be the reflection point of F with respect to AE. Prove that $\triangle B_1D_1F_1\sim BDF$.