

2016 Kosovo Team Selection Test
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by dangerousliri

- 1 Solve equation in real numbers

$$\sqrt{x + \sqrt{4x + \sqrt{16x + \sqrt{\dots + \sqrt{4^n x + 3}}}} - \sqrt{x}} = 1$$

- 2 Show that for all positive integers $n \geq 2$ the last digit of the number $2^{2^n} + 1$ is 7.

- 3 If quadratic equations $x^2 + ax + b = 0$ and $x^2 + px + q = 0$ share one similar root then find quadratic equation for which has roots of other roots of both quadratic equations.

- 4 It is given the function $f : \mathbb{R} \rightarrow \mathbb{R}$ for which $f(1) = 1$ and for all $x \in \mathbb{R}$ satisfied

$$f(x + 5) \geq f(x) + 5 \text{ and } f(x + 1) \leq f(x) + 1$$

If $g(x) = f(x) - x + 1$ then find $g(2016)$.

- 5 Let be ABC an acute triangle with $|AB| > |AC|$. Let be D point in side AB such that $\angle ACD = \angle CBD$. Let be E the midpoint of segment BD and S let be the circumcenter of triangle BCD . Show that points A, E, S and C lie on a circle.