Danube Mathematical Competition 201

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by parmenides51, drEdrE

1 Let $ABCM$ be a quadrilateral and D be an interior point such that $ABCD$ is a parallelogram. It is known that $\angle AMB = \angle CMD$. Prove that $\angle MAD = \angle MCD$.

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2 Let S be a set of positive integers such that: min lcm (x, y) : x, y S, $x \neq y$ $\geq$ 2 + max S.
Prove that $\sum_{x\in S}\frac{1}{x} \leq \frac{3}{2}$.

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3 Determine all positive integer numbers n satisfying the following condition:
the sum of the squares of any n prime numbers greater than 3 is divisible by n.

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4 Given a positive integer number n, determine the maximum number of edges a triangle-free Hamiltonian simple graph on n vertices may have.
